

Economics of the Market

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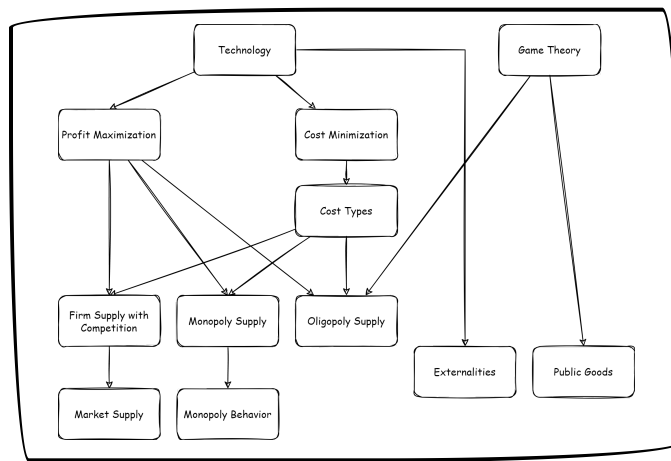
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1. Markets, Strategies, and Firms

1.1. Context

- The majority of economic transactions take place through markets. Newspaper articles focus on developments in a plethora of markets daily.
- Although the concept is familiar to most of us through mass media, markets have a myriad of flavors, each one with particular characteristics.
 - What is a market from an economic perspective?
 - Why are economists and politicians so interested in how markets are organized?
 - How do markets contribute to the organization of production?

1.2. Course Structure Overview



1.3. Lecture Structure and Learning Objectives

Structure

- Jonathan Lebed (Case Study)
- Markets, Strategies, and Firms from an Economic Perspective.
- Current Field Developments.

Learning Objectives

- Explain what a market is from an economic perspective.
- Describe with examples the main typology of markets.
- Illustrate the organizational differences within and across firms.
- Give an overview of the fundamental market structures and competition strategies.

1.4. Jonathan Lebed

- Lebed is a former stock market trader.
- He was raised in New Jersey, US.
- He was prosecuted by the US Securities and Exchange Commission (SEC) for stock manipulation.
- Lebed reached an out-of-court settlement with SEC in 2000; He was 15 years old.

1.4.1. The SEC Prosecution

- Lebed is the first minor ever prosecuted for stock-market fraud.
- Lebed tools were
 - an America Online (AOL) internet connection,
 - an E*trade account, and
 - four email accounts in Yahoo Finance Message Boards.
- The SEC accused him of making his money through a *pump and dump* strategy.

1.4.2. Timeline

- Shortly after his **11-th** birthday Jonathan opened an account with America Online.
- He started building a website about pro-wrestling.
- At the age of 12, he invested **\$8000** (via his father) in the stock market, taken from a bond his parents gave him at birth.
- He started building an amateur investor website `www[dot]stock-dogs[dot]com`"
- At 14, the SEC charged him with civil fraud.
- His mother closed his trading account.
- His father opened another account for him!



1.4.3. The Settlement

- Lebed forfeited **\$285000** in profit and interest he had made on **11** trades.
- He has never admitted any wrongdoing.
- He kept close to **\$500000** in profit.

1.4.4. Everybody is Manipulating the Market

People who trade stocks, trade based on what they feel will move and they can trade for profit. Nobody makes investment decisions based on reading financial filings. Whether a company is making millions or losing millions, it has no impact on the price of the stock. Whether it is analysts, brokers, advisors, Internet traders, or the companies, everybody is manipulating the market. If it wasn't for everybody manipulating the market, there wouldn't be a stock market at all...

Jonathan Lebed, statement to his lawyer, ([Lewis 2001](#))

1.5. Markets

- The first markets were locations where people gathered to purchase and sell commodities.
- Today, thinking in such terms is way too restrictive to describe markets accurately.
- In financial markets, contracts are traded instead of commodities.
- In labor markets, services are traded.
- Internet markets do not have physical manifestations.

1.5.1. Markets in Economics

- Economics studies the interactions of economic entities; namely firms, households, governments, and other organizations.
- How do these agents interact?
- A **market** is a coordination mechanism *conveying information via prices* among economic entities to organize production and allocate output.

1.5.2. Types of Markets

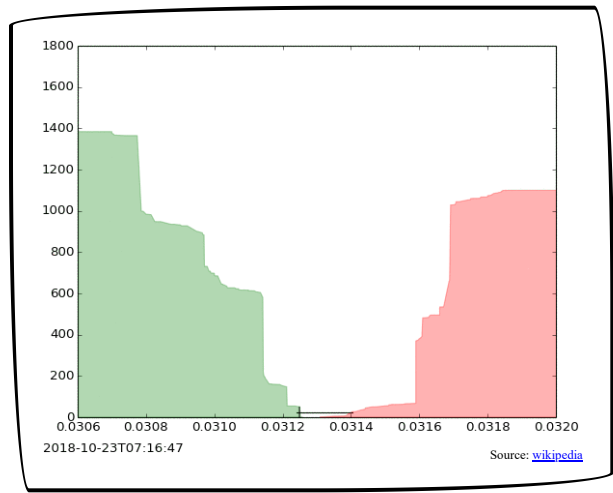
- **Physical markets** are markets that manifest in specific geographical locations. **Non physical markets** are markets that are not physical. (e.g., a supermarket vs. a labor market.)
- **Retail markets** are final goods and services markets sold for consumption. **Business markets** are final or intermediate goods and services markets sold from one firm to another. (e.g., a fish market vs. a tractor market.)
- **Financial markets** are markets in which financial instruments and contracts are traded. **Non financial markets** are markets in which commodities and services are traded. (e.g., a stock exchange vs. a grocery store.)
- **Authorized markets** are markets where the trade of commodities occurs via channels authorized by the manufacturers. **Unauthorized markets** (or **gray markets**) are markets with channels that the manufacturer does not recognize. (e.g., US version video games market in the US vs. Japanese version video games in the US)
- **Legal markets** are markets in which commodities and services are traded according to the governing set of rules. **Illegal markets** (or **black markets**) are markets in which transactions have some aspect of non compliance with the governing rules. (e.g., car market vs. illegal drug market)

1.5.3. Limits of Markets

- How inclusively a market is defined entails a level of arbitrariness.
- An electric automobile sold in Frankfurt belongs in
 - Frankfurt's automobile market
 - Hessen's automobile market
 - Germany's automobile market
 - Germany's electric automobile market
 - European automobile market
- The definition of what a market includes is crucial in legal cases concerning competition law.

1.5.4. Market Forces

- Two market forces comprise markets.
 - Demand (Microeconomics I)
 - Supply (This course).
- Prices interlink demand and supply in markets.
- These two market forces are the main analytical tools in the economic analysis of markets.
- Markets are defined by the intentions to trade, not only the actual trades taking place.
- Example: Stock Exchange order books.



1.5.5. Market Failures

- Coordination mechanisms based on prices do not always exist.
- Some commodities and services have special characteristics that
 - either prevent efficient coordination based on prices,
 - or even lead to a complete production shutdown!
- A **market failure** is a situation in which the price-based coordination of economic agents is impossible or inefficient.

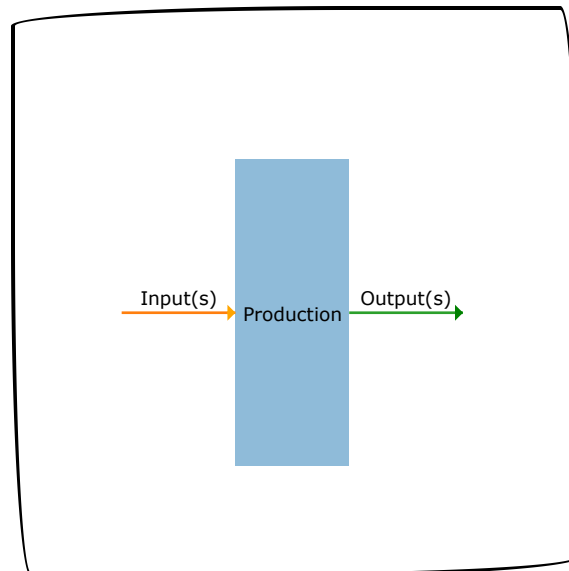
1.6. Firms

- A **firm** is a legal entity that signs contracts with its suppliers, distributors, employees, and often customers ([Chandler 1992](#)).

1.6.1. Firms in Economics and Business

- Looking within the firm (Economics of the Firm):
 - A firm is treated as an administrative entity in which a team of managers is needed to coordinate and monitor its activities and the division of labor.
 - How do managers and workers interact?
- Looking across firms (This course):
 - A firm becomes a pool of know-how, physical, and financial capital.
 - It uses its resources to produce output.
 - For-profit firms are the primary instruments in capitalist economies for producing and distributing commodities and services.

1.6.2. Production as a Black-box



1.7. Strategies

- The collection of firms in a market forms market supply.
 - This collection can consist of exactly one firm (monopoly).
 - It can include a small number of firms (oligopoly).
 - It can include a very large (formally infinite) number of firms (perfect competition).

1.7.1. Competition

- Firms may compete by choosing prices.
- They may follow price discrimination strategies.
- They may compete by choosing quantities.
- They may differentiate their products to stand out.

1.8. Current Field Developments

- The transaction cost theory of the firm focusing on the firm's relation to the market has started developing in the 1930s.
- Managerial and behavioral theories of the firm focusing on internal organization have started developing in the 1960s.
- Industrial organization is an economic field that builds on the theory of the firm and examines the market structure and the relationships among firms.

1.9. Concise Summary

- For economics, markets are the primary coordination mechanism of production and allocation.
- There is a rich typology of markets based on their characteristics (physicality, legality, etc.).
- When studying competition and market structure, it is convenient to abstract from organizational aspects and consider the firm as a black box.
- Based on this, various firm models can be used to analyze competition in the market (monopoly, oligopoly, perfect competition, etc.)
- Firms use various strategies based on prices and quantities to compete in a market.

1.10. Further Reading

- Varian ([2010, chap. 1](#))
- Belleflamme and Peitz ([2010, secs. 1.1-1.4, 2.1.1-2.1.4](#))
- Chandler ([1992](#))

1.11. Exercises

1.11.1. Group A

1. Consider a market with demand $d(p) = 100 - 4p$ and supply $s(p) = 40 + 2p$.
 - a. Calculate the demanded and supplied quantities for $p = 2$. Does the market clear? Does it have a shortage or a surplus?
 - b. Calculate the market clearing price.
2. Consider a market with demand $d(p) = 250/p$ and supply $s(p) = 10p$.
 - a. Calculate the market clearing price (assume that the price cannot be negative).
 - b. Calculate the price elasticity of demand.
 - c. Calculate the price elasticity of supply.
3. Consider a market with demand $d(p) = 22 - 3p$ and supply $s(p) = 10 + p$.
 - a. Calculate the market clearing price.
 - b. Calculate the price elasticity of demand at the market clearing price.
 - c. Calculate the price elasticity of supply at the market clearing price.

1.11.2. Group B

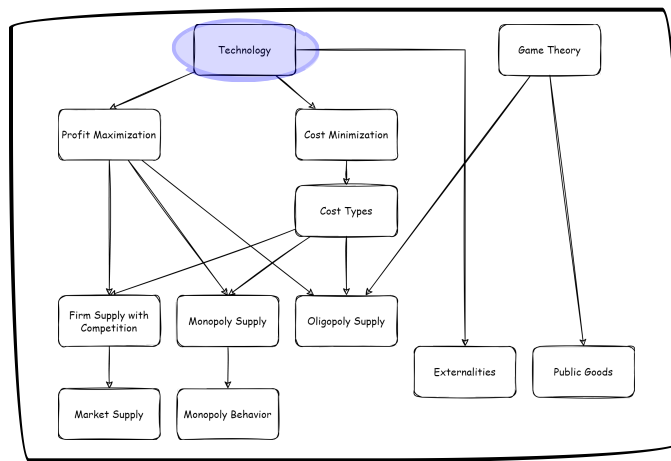
1. Consider a market with demand $d(p) = \alpha - \beta p$ and supply $s(p) = \gamma + \delta p$, where all Greek letters are positive parameters.
 - a. Calculate the market clearing price.
 - b. Calculate the price elasticity of demand at the market clearing price.
 - c. Calculate the price elasticity of supply at the market clearing price.

2. Technology

2.1. Context

- The sustained increase in living standards is primarily due to technological progress (e.g., the Industrial Revolution, the Information Revolution, etc.).
- However, not all technological changes favor everyone in the economy.
 - How is technological progress incorporated into firm production?
 - How does it affect the scaling of production?
 - How does it affect the allocation of production resources?

2.2. Course Structure Overview



2.3. Lecture Structure and Learning Objectives

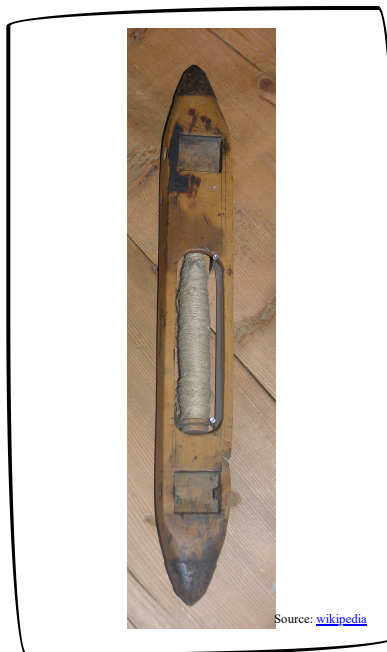
Structure

- The Flying Shuttle (Case Study)
- Basic Concepts
- A Cobb-Douglas Example
- Current Field Developments

Learning Objectives

- Explain how economists describe technological progress in modern applications.
- Describe the relationship between production inputs induced by various technologies.
- Analyze the effect of scaling inputs on production output.

2.4. The Flying Shuttle



2.4.1. Textile Technology

- Textile production has two main sub-processes:
 - Spinning
 - Weaving

2.4.2. Spinsters

- Fibers are "spun" together to form yarn.
- Performed mainly by women in the pre-industrial revolution era.



2.4.3. Weavers

- Two yarns were vertically interlaced to form cloth.
- Performed mainly by men in the pre-industrial revolution era.
- Large cloth required the coordination of two weavers.



2.4.4. The Industrial Revolution Leap

- John Kay invented the flying shuttle in 1733.
- It greatly increased weaving automation.
- No coordination between weavers was needed anymore.



2.4.5. What Was the Impact of the Flying Shuttle?

- Weaving productivity increased →

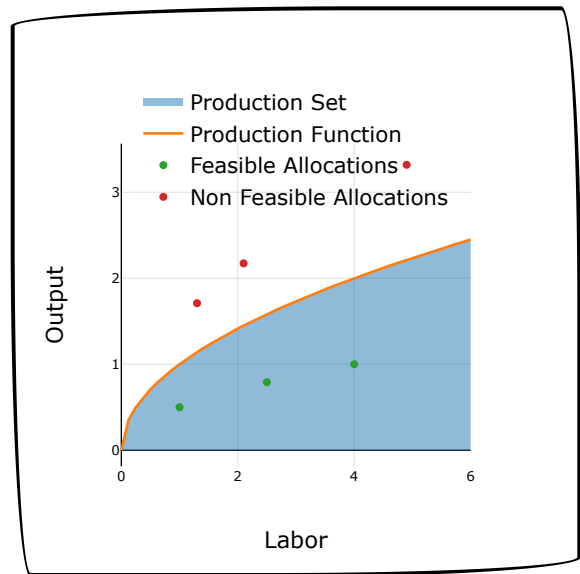
- Cloth production increased →
- Demand for weavers decreased (technological unemployment) →
- Yarn demand increased →
- Spinsters could not keep up! →
- The new technology changed the allocation of capital and labor in textile production!
- (And Spinning jenny was invented a bit later)

2.5. Production Output and Factors

- The **production output** is the result of a production process (Typically a commodity or a service produced by a firm).
- The **production factors** (or **input factors**) are the resources used in a production process.
- Which are the most common production factors?
 - Labor
 - Human Capital
 - Physical Capital
 - Financial Capital
 - Land
- How do we measure the production factors and output?
 - Units per time (e.g., hours worked per week, cars produced per year, etc.).

2.6. Technological Feasibility

- A **feasible allocation** is a combination of input factors and output such that the output can be produced using the inputs for a given technology.
- The set of all feasible combinations of inputs and output for a given technology is called **production set**.



2.7. Production Function

- A function that transforms amounts of input factors to the maximum amount of output that can be produced for a given technology is called a **production function**.

- **Cobb-Douglas production:**

$$f(K, L) = 10K^{0.5}L^{0.5}$$

- **Fixed proportions production:**

$$f(K, L) = \min \{K, L\}$$

- **Perfect substitutes production:**

$$f(K, L) = 3K + 2L$$

- **Root production:**

$$f(L) = 5\sqrt{L}$$

2.7.1. Production Properties

- A production function is **monotonic** if output increases whenever the input factors increase.
- A production technology exhibits the **free disposal** property if inputs can be discarded at no cost.

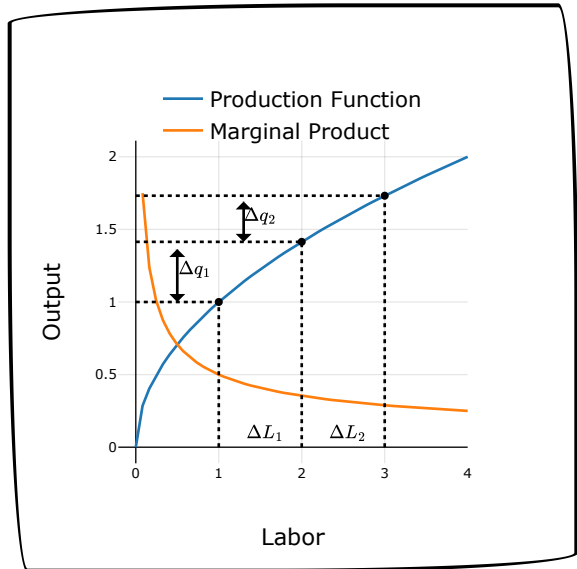
2.8. Marginal Product

- How does output change when some input changes?
- The **marginal product** is the rate of change of the production output with respect to an input factor.

- For a production function f , the marginal product is the derivative f' .

2.8.1. "Law" of Diminishing Marginal Product

- For many production technologies, expanding production becomes increasingly more difficult. As production increases, the marginal products of the production factors diminish. This property is known as the **law of diminishing marginal product**.



2.8.2. Marginal Product Example

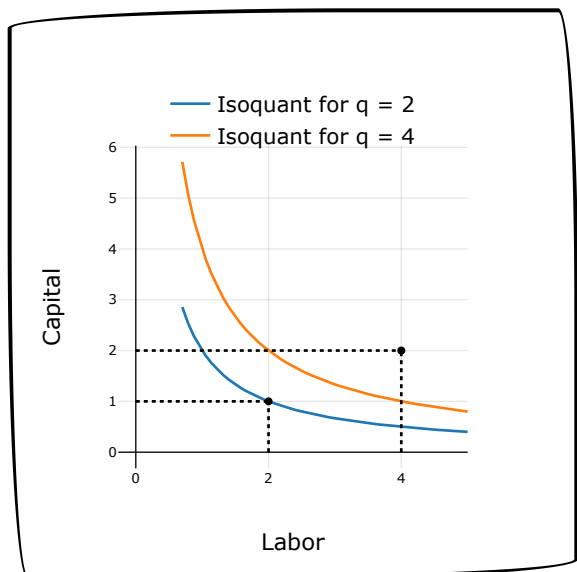
- If $f(L) = 5\sqrt{L}$,
- then $MP(L) = f'(L) = \frac{5}{2} \frac{1}{\sqrt{L}}$

2.9. Returns to Scale

- What happens to production if we double all the input factors?
 - We check what happens to the isoquant.
- An **isoquant** is a locus of points showing all the technologically efficient ways of combining production factors to produce a fixed level of output.

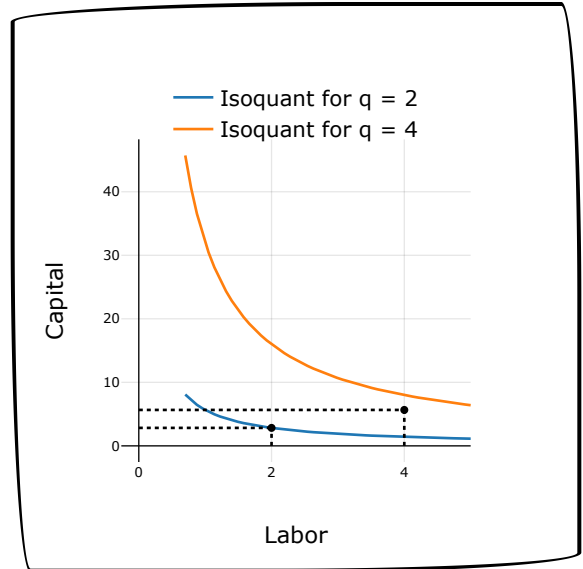
2.9.1. Increasing Returns to Scale

- A production function exhibits **increasing returns to scale** if its output is more than doubled whenever its input factors are doubled.



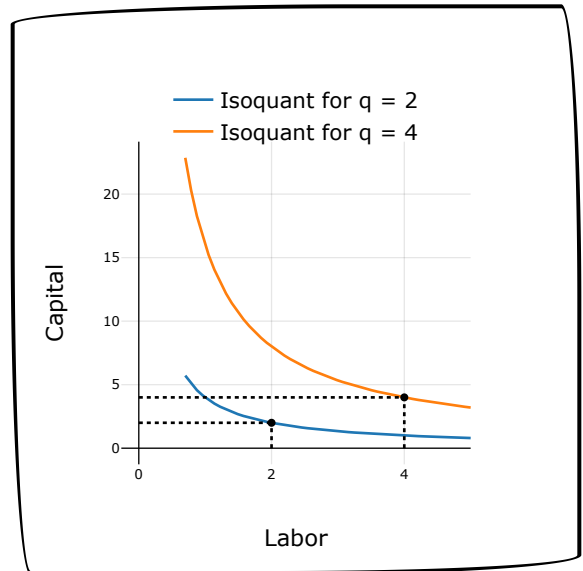
2.9.2. Decreasing Returns to Scale

- A production function exhibits **decreasing returns to scale** if its output is less than doubled whenever its input factors are doubled.



2.9.3. Constant Returns to Scale

- A production function exhibits **constant returns to scale** if output doubles whenever its input factors are doubled.



2.9.4. Returns to Scale Example

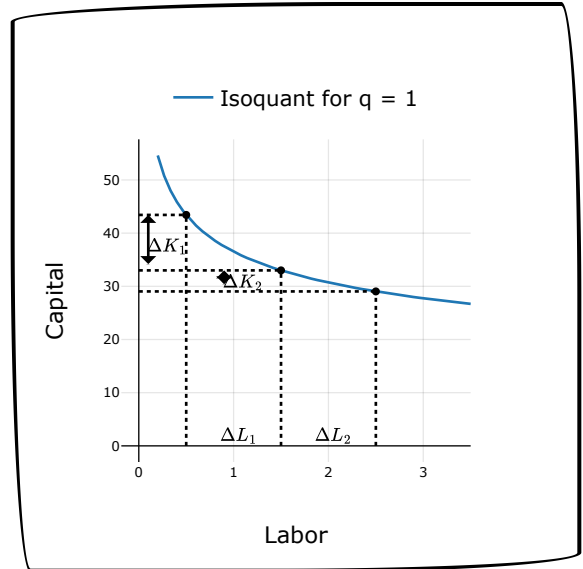
- Suppose that $f(K, L) = 10K^\alpha L^\beta$.
- Then
 - $\alpha + \beta > 1$,
 - decreasing returns to scale if $\alpha + \beta < 1$, and
 - constant returns to scale if $\alpha + \beta = 1$.

2.10. Marginal Rate of Technical Substitution

- How much more capital do we need to keep the output constant if we decrease labor by one unit?
 - We check what happens to the marginal rate of technical substitution (MRTS).
- The **marginal rate of technical substitution** is the rate of change of an input factor with respect to changes of another input factor while production is kept constant.

2.10.1. Diminishing Marginal Rate of Technical Substitution

- For most (all?) production technologies, if one production factor (e.g., labor) is decreased, another production factor (e.g., capital) has to increase to keep production constant. Thus, in most cases, MRTS is decreasing.



2.10.2. Marginal Rate of Technical Substitution Example

- Suppose that $f(K, L) = K^{0.4}L^{0.6}$.
- Then f has
 - $MP(K; L) = \frac{\partial f(K, L)}{\partial K} = 0.4K^{-0.6}L^{0.6}$,
 - $MP(L; K) = \frac{\partial f(K, L)}{\partial L} = 0.6K^{0.4}L^{-0.4}$, and
 - $MRTS(K, L) = \frac{MP(K; L)}{MP(L; K)} = \frac{2}{3} \frac{L}{K}$.

2.11. Current Field Developments

- Technology affects productivity
- How can we measure productivity?
 - Take log in the Cobb-Douglas production function.

$$\log q = \log A + \alpha \log K + \beta \log L$$

- If we regress $\log q$ on $\log K$ and $\log L$ (which we observe), the estimation constant gives us an estimate of $\log A$.
- Initially performed at a macroeconomic level.
- More advanced methods try to estimate A at an industry (even gender) specific level.

2.12. Concise Summary

- Technology governs
 - how economical is a production process,
 - how different production factors substitute each other (MRTS), and
 - how easily production can be scaled (returns to scale).
- Different production functions describe different technologies.
- In many cases, production output is increasingly more difficult to produce ("Law" of diminishing marginal product).
- In most cases, reducing one input factor (e.g., labor) requires increasing another factor (e.g., capital) to keep output constant (Diminishing MRTS).

2.13. Further Reading

- Varian ([2010, chap. 19](#))
- CORE Team ([2017, secs. 2.4, 7.2](#))

2.14. Mathematical Details

2.14.1. Production Function

Suppose that some production technology induces a production set $\mathcal{Q}$. For single output production technologies, the production function is defined as the maximum amount of output that is feasible for a given combination of input factors. Thus, with one factor

$$f(x) = \max \{q \geq 0: (x, q) \in \mathcal{Q}\},$$

and with two factors

$$f(x_1, x_2) = \max \{q \geq 0: (x_1, x_2, q) \in \mathcal{Q}\}.$$

2.14.2. Some Usual Production Functions

1. Root Production Function (1D Factor Space)

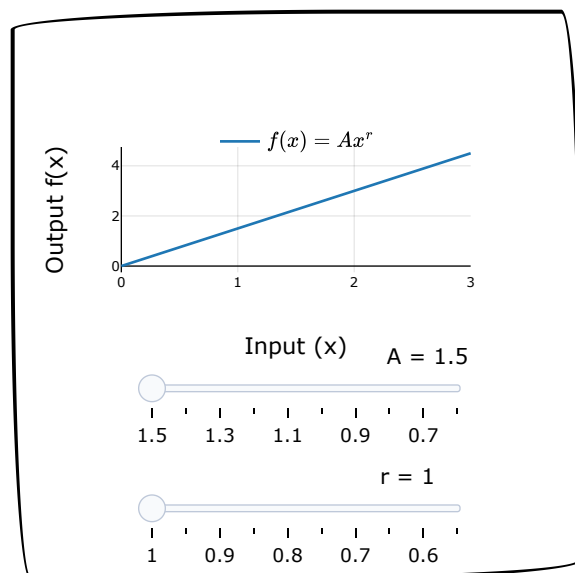
Consider the production set

$$\mathcal{Q} = \{(x, q) \in \mathbb{R}_{\geq 0}^2 : 0 \leq q \leq Ax^r\}$$

where $A > 0$, $r > 1$. The production function resulting from $\mathcal{Q}$ is

$$f(x) = Ax^r.$$

The parameter A scales production uniformly, while r controls the production function's curvature.



2. Constant Elasticity of Substitution Production Function (nD Factor Space)

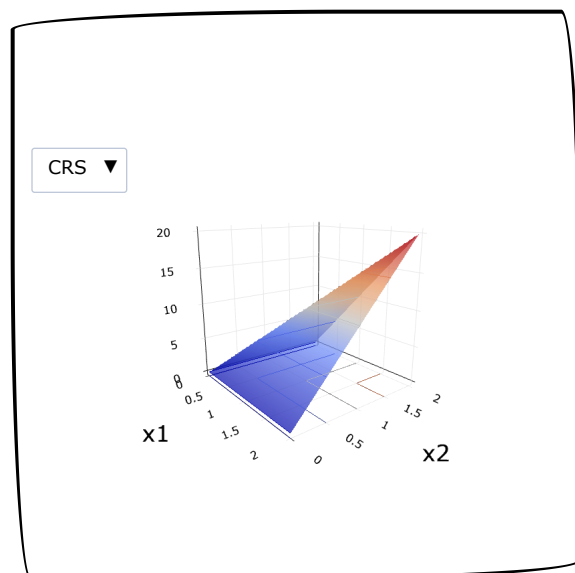
The constant elasticity of substitution production function is given by

$$f(x_1, x_2) = A(\alpha x_1^\rho + (1 - \alpha)x_2^\rho)^{\frac{1}{\rho}}$$

where $A > 0$, $r > 0$, $\alpha \in (0, 1)$, $\rho \leq 1$. Essentially, production output is modeled as a generalized mean of input factors. The parameter A is interpreted as productivity, r controls the returns to scales, α controls the shares of the factor in production, and ρ is the elasticity of substitution between the factors. Three special cases can be obtained by changing the value of ρ .

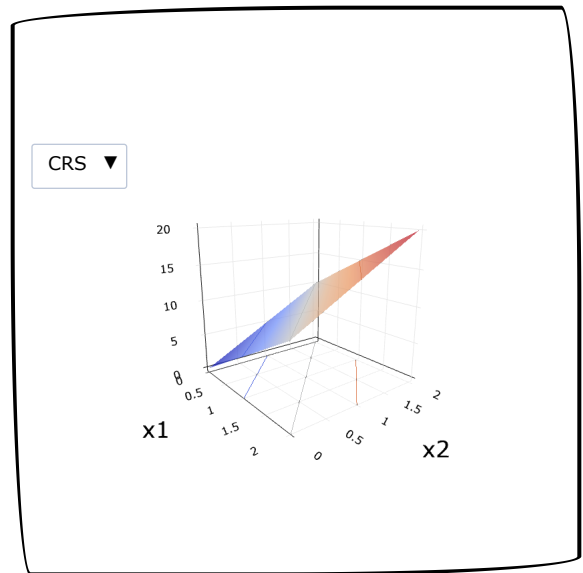
a. Fixed Proportions

For $\rho \rightarrow -\infty$, we get $f(x_1, x_2) \rightarrow A \min\{x_1, x_2\}^r$



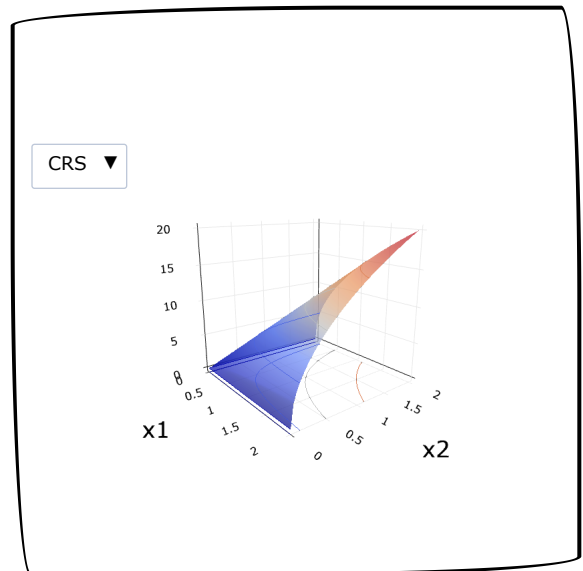
b. Perfect Substitutes

For $\rho = 1$, we get $f(x_1, x_2) = A(\alpha x_1 + (1 - \alpha)x_2)^r$



c. Cobb-Douglas

Lastly, for $\rho \rightarrow 0$, we get $f(x_1, x_2) \rightarrow Ax_1^\alpha x_2^{(1-\alpha)r}$



2.14.3. Returns to Scale

1. Increasing Returns to Scale

A production function f exhibits increasing returns to scale if, for any $t > 1$, it holds

$$f(tx_1, tx_2) > tf(x_1, x_2).$$

Output is scaled more than the amount with which the input factors are scaled.

2. Decreasing Returns to Scale

A production function f exhibits decreasing returns to scale if, for any $t > 1$, it holds

$$f(tx_1, tx_2) < tf(x_1, x_2).$$

Output is scaled less than the amount with which the input factors are scaled.

3. Constant Returns to Scale

A production function f exhibits constant returns to scale if, for any $t > 0$, it holds

$$f(tx_1, tx_2) = tf(x_1, x_2).$$

Output is scaled exactly as the amount with which the input factors are scaled.

2.14.4. What Happens when some Factors are Fixed?

Suppose that the original production function has two factors, namely $f_1(x_1, x_2)$. Due to some physical or legal constraint, the firm cannot change x_2 , which is kept constant equal to $\bar{x}_2$. This can happen if the time frame of interest is too short to allow the firm to adjust x_2 . The resulting production

is

$$f_2(x_1) := f_1(x_1, \hat{x}_2).$$

2.14.5. Usually Assumed Properties

1. Monotonicity

A production function is monotonic, if whenever inputs increase, then the outputs do not decrease.

$$x_1 \geq x_2 \implies f(x_1) \geq f(x_2)$$

If f is differentiable, then $f' \geq 0$, if and only if f is monotonic.

2. Free Disposal

A technology is characterized by free disposal if inputs can be discarded at no cost. If a technology exhibits free disposal, its production function is monotonic (why?).

3. Production Set Convexity

A technology is convex when mixtures of feasible combinations are also feasible. If technology is convex, then the corresponding production function is concave. A (production) function f is concave if for any $\lambda \in [0, 1]$ we have

$$f(\lambda x_1 + (1 - \lambda)x_2) \geq \lambda f(x_1) + (1 - \lambda)f(x_2).$$

If f is differentiable, then $f'' \leq 0$ if and only if f is concave.

2.14.6. Marginal Product

The marginal product is the rate of change of the production output with respect to an input factor. For single-dimensional factor spaces, by the definition of the derivative, we have

$$f'(x_0) = \lim_{x \rightarrow x_0} \frac{f(x) - f(x_0)}{x - x_0}.$$

For multidimensional factor spaces, by the definition of the partial derivative, we have

$$\frac{\partial f}{\partial x_0}(x_0, x_1) = f_{x_0} = \lim_{x \rightarrow x_0} \frac{f(x, x_1) - f(x_0, x_1)}{x - x_0}.$$

The marginal product of factor x_i is diminishing when f_{x_i} is decreasing. If f_{x_i} is differentiable, then the marginal product of factor x_i is diminishing if and only if $f_{x_i x_i} \leq 0$. For a single factor production function, the last condition is true if and only if the function is concave.

2.14.7. Marginal Rate of Technical Substitution

For a fixed level of output, say $q = f(x_1, x_2)$, we can think of the implicit mapping

$$g(x_1) = \{x_2 \in \mathbb{R}: q = f(x_1, x_2)\}.$$

This mapping gives us the isoquant for q . Under some invertibility and smoothness conditions that most commonly used production functions satisfy, the isoquant is differentiable. The opposite of its derivative typically denoted as

$$-\frac{dx_2}{dx_1} = \text{MRTS}(x_1, x_2) = \frac{\partial f(x_1, x_2)/\partial x_1}{\partial f(x_1, x_2)/\partial x_2},$$

is the rate of change of x_2 with respect to x_1 given that production is kept constant at a level q . This rate of change is called the marginal rate of technical substitution.

2.14.8. Elasticity of Substitution

The elasticity of substitution is defined by

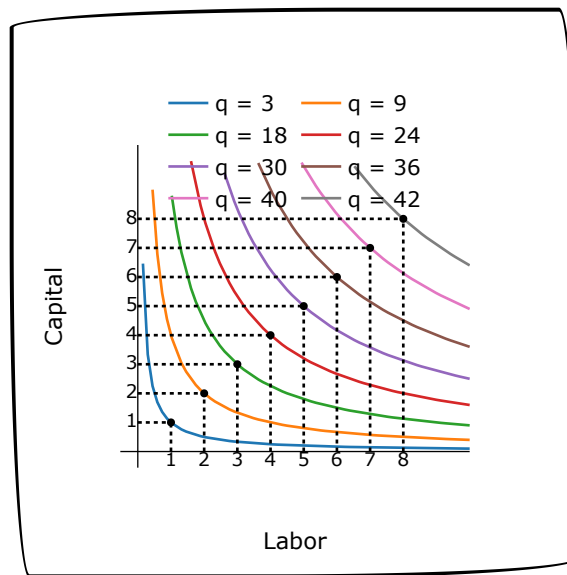
$$e(x_2, x_1) = \left(\frac{d \ln \text{MRTS}(x_1, x_2)}{d \ln(x_2/x_1)} \right)^{-1}.$$

It constitutes a measure of the substitutability between input factors in a production function.

2.15. Exercises

2.15.1. Group A

1. The following graph plots the isoquants of a production function for various levels of output. Determine the regions where the production function exhibits constant, increasing, and decreasing returns to scale.



2. Consider the family of production sets

$$\mathcal{Q}^r = \{(x, q) \in \mathbb{R}_{\geq 0}^2 : 0 \leq q \leq x^r\} \quad (0 < r < \infty).$$

- Plot in a single graph the production functions for $0 < r_1 < 1 = r_2 < r_3$.
- Sketch the production set for a fixed $r < 1$.
- Sketch the production set for a fixed $r > 1$.
- For which values of r the marginal product of the production function corresponding to $\mathcal{Q}^r$ is diminishing / constant / increasing?

2.15.2. Group B

- Consider the production function $f(x_1, x_2) = x_1^\alpha x_2^\beta$ for parameters $\alpha, \beta > 0$. For which values of α and β does f exhibit decreasing / constant / increasing returns to scale?

2.15.3. Group C

- Show that the elasticity of the constant elasticity production function is constant and equal to $(1 - \rho)^{-1}$.
- Consider the constant elasticity of substitution function

$$f(x_1, x_2) = A(\alpha x_1^\rho + (1 - \alpha)x_2^\rho)^{\frac{1}{\rho}}.$$

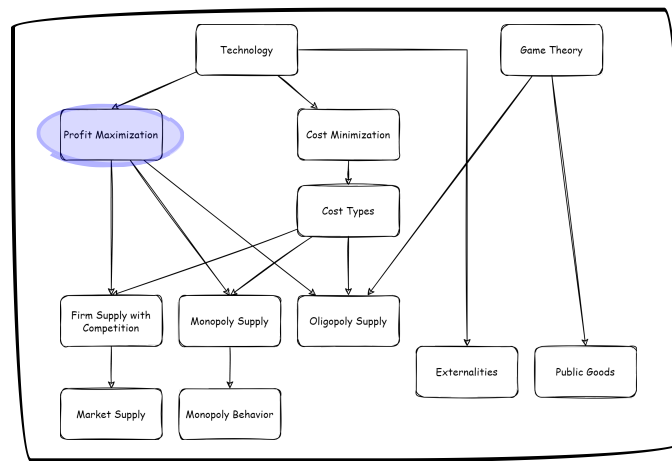
- Show that for $\rho \rightarrow -\infty$ f converges to the fixed proportion production function $f(x_1, x_2) \rightarrow A \min\{x_1, x_2\}^r$.
- Show that for $\rho = 1$ f reduces to the perfect substitutes production function $f(x_1, x_2) = A(\alpha x_1 + (1 - \alpha)x_2)^r$.
- Show that for $\rho \rightarrow 0$ f converges to the Cobb-Douglas production function $f(x_1, x_2) \rightarrow A x_1^{\alpha r} x_2^{(1-\alpha)r}$.

3. Profit Maximization

3.1. Context

- Profit maximization is ubiquitous in the economic theory of firms' behavior.
- In practice, many successful managers often divert from following this objective.
 - Why is, then, profit maximization used so often in theory?
 - Can we still learn something from it?
 - How does it work?

3.2. Course Structure Overview



3.3. Lecture Structure and Learning Objectives

Structure

- The Bloody ROI (Case Study)
- Basic Concepts
- Examples
- Current Field Developments

Learning Objectives

- Explain why profit maximization has such a prominent role in modern applications.
- Explain apparent discrepancies between economic theory and business practice.
- Describe the solution to the profit maximization problem with a single input.
- Describe the solution to the profit maximization problem with multiple inputs.
- Analyze the relationship between demanded input factors and prices.

3.4. The Bloody ROI

Mr. Cook replied –with an uncharacteristic display of emotion– that a return on investment (ROI) was not the primary consideration on such issues. "When we work on making our devices accessible by the blind," he said, "I don't consider the bloody ROI." It was the same thing for environmental issues, worker safety, and other areas that don't have an immediate profit. The company does "a lot of things for reasons besides profit motive. We want to leave the world better than we found it."

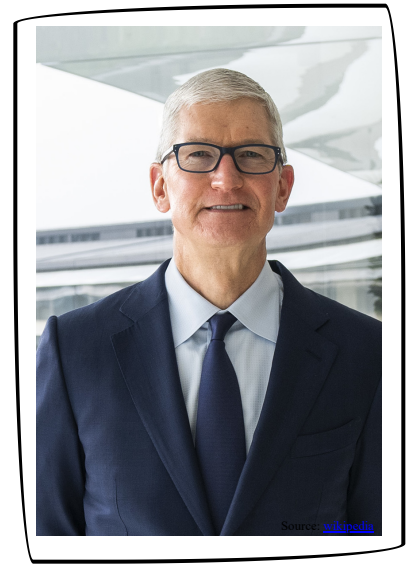
Why Tim Cook Doesn't Care About 'The Bloody ROI', ([Denning 2014](#))

3.4.1. Apple's 2014 Shareholder Meeting

- One shareholder is the National Center for Public Policy Research (NCPPR) investor group.
 - A conservative think tank.
 - Does not own much stock in Apple (not in the top stockholders).
- Apple had many programs targeting environmental sustainability.
- NCPPR pushed a proposal about disclosing the costs of such sustainability programs.
 - Argued for more transparency in actions towards the "amorphous concept of environmental sustainability".
 - Rejected by Apple shareholders, with only **3%** voting in favor.

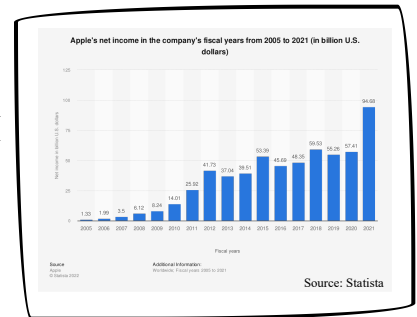
3.4.2. The Q & A Incident

- Tim Cook was asked two questions by the NCPPR representative in the Q&A.
 - Whether these "green actions" that the company had adopted were good for profitability?
 - Whether the company would commit to only taking actions that were good for profitability?
- Reports say that an angry Tim Cook included in his reply that "If you want me to do things only for ROI reasons, you should get out of this stock."



3.4.3. Doesn't Tim Cook Care About Apple's Profits?

- This was not the gist of Tim Cook's answer!
- Apple was immensely profitable at the time of the incident!
- Still, many of its individual activities produce no profits.
- The 2020 McKinsey survey indicated that 33% of managers experienced trade-offs between purpose and profit, while 72% percent of the employees wanted purpose to be more central ([Gast et al. 2020](#)).



3.4.4. Why is Profit Maximization So Prominent in Theory?

- In practice, only chasing profits can lead to tunnel vision.
- Short-sighted strategies do not always have the desired effect in the long-run.
- Sometimes companies pursue other strategic goals (e.g., social and environmental responsibility).
- Antiwar campaigns, environmental programs, and free software updates might not be profitable.
- However, they help create a customer ecosystem and build customer loyalty.
- This, in turn, can lead to sustainable, greater profits in the long-run.
- Nonetheless, ignoring profits for too long can lead to insolvency!
- Where is the golden ratio? Finding it distinguishes good and bad managers!

3.5. Profit Maximization

- The **profit maximization** assumption is the most common approach in the economic modeling of the behavior of firms.
- Firms choose prices, inputs, and outputs to maximize profits.
- Profit = Revenue - Cost

$$\pi(q, K, L) = \underbrace{pq}_{\text{Revenue}} - \underbrace{(rK + wL)}_{\text{Cost}}$$

- Profits are usually measured in monetary units per time (e.g., Euros per week, Dollars per month, etc.).

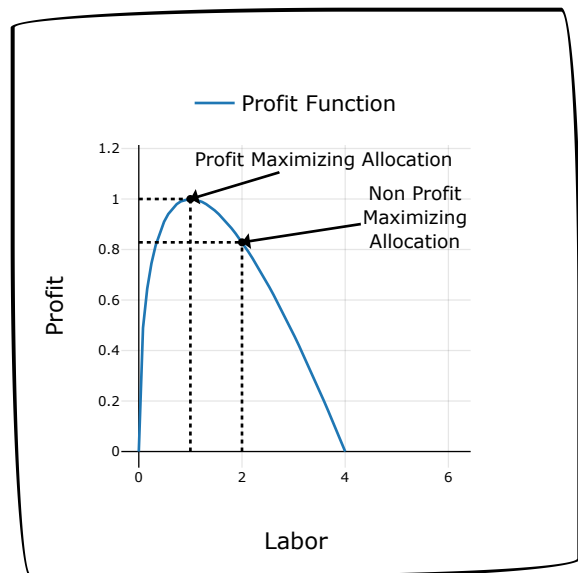
3.5.1. Example: Profits with a Root Production Function

- Suppose production is only based on labor, i.e.,

$$q = \sqrt{L}.$$

- Then, profit is

$$\pi(L) = p\sqrt{L} - wL$$



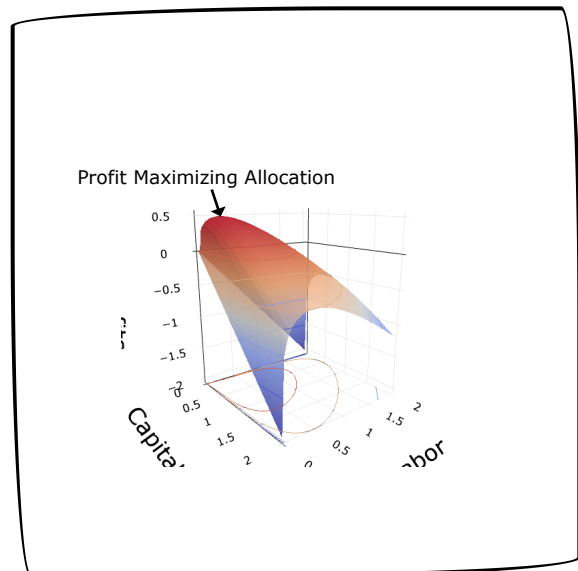
3.5.2. Example: Profits with a Cobb-Douglas Production Function

- Suppose production is given by

$$q = K^{1/4}L^{1/4}.$$

- Then, profit is

$$\pi(K, L) = pK^{1/4}L^{1/4} - rK - wL$$



3.6. Scaling Profits

- What happens to profit if we double all the input factors?
 - We check what happens to the isoprofit curve.
- An **isoprofit** curve on the output-input plane is a locus of points showing all the technologically efficient ways of combining production factors resulting in equal profits.

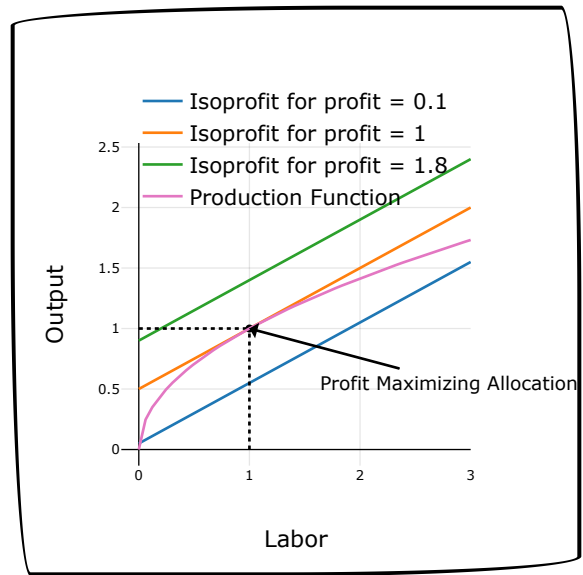
3.6.1. Example: Isoprofit and Profit Maximization

- Suppose that profit is

$$\pi = pq - wL.$$

- Fix profit at $\hat{\pi}$ and solve for q

$$q = \frac{\hat{\pi}}{p} + \frac{w}{p}L$$



3.7. Factor Demand and Inverse Factor Demand

- Profit maximization induces some interesting relationships between input factors and prices.

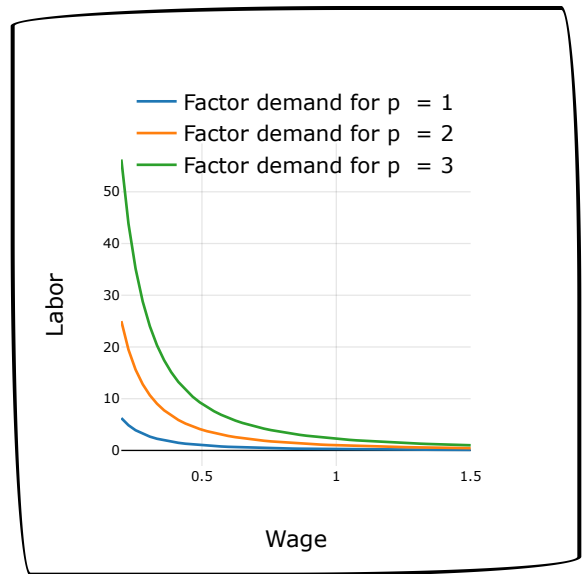
3.7.1. Factor Demand

How much from each factor would a profit maximizing firm like to use?

- We check the factor demand function.
- The **factor demand** is a function that gives the profit maximizing input factor quantity for given input and output prices.

3.7.2. Example: Factor Demand for Root Production

$$L(w) = \frac{1}{4} \frac{p^2}{w^2}$$



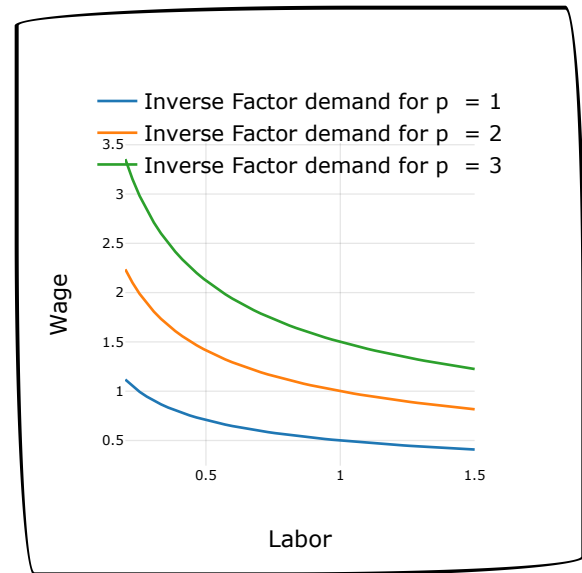
3.7.3. Inverse Factor Demand

For which price level is an input factor choice profit maximizing?

- We check the inverse factor demand function.
- The **inverse factor demand** is a function that gives the input price factor for which a factor choice maximizes profits.

3.7.4. Example: Inverse Factor Demand for Root Production

$$w(L) = \frac{1}{2} \frac{p}{\sqrt{L}}$$



3.8. Revealed Profit Maximization Inequality

- Suppose we observe choices (q_1, K_1, L_1) and (q_2, K_2, L_2) of a profit maximizing firm on dates **1** and **2**.
- Since (q_1, K_1, L_1) is profit maximizing under prices (p_1, r_1, w_1) , then

$$p_1 q_1 - r_1 K_1 - w_1 L_1 \geq p_1 q_2 - r_1 K_2 - w_1 L_2.$$

- Since (q_2, K_2, L_2) is profit maximizing under prices (p_2, r_2, w_2) , then

$$p_2 q_2 - r_2 K_2 - w_2 L_2 \geq p_2 q_1 - r_2 K_1 - w_2 L_1.$$

- Adding these two inequalities gives

$$\Delta p \Delta q - \Delta r \Delta K - \Delta w \Delta L \geq 0.$$

3.8.1. Interpretation

- If we find combinations of inputs, outputs, and prices for which the resulting inequality does not hold, then the firm was not maximizing profits on at least one of the dates.
- If all combinations of inputs, outputs, and prices in our data satisfy the resulting inequality, then we cannot reject the possibility of profit maximizing behavior.

3.8.2. Supplied Quantities and Prices

- If the prices of input factors remain constant (i.e., $\Delta r = \Delta w = 0$), then

$$\Delta p \Delta q \geq 0.$$

- Changes in output quantities should have the same sign as changes in output prices.

3.8.3. Factor Demanded Quantities and Prices

- If the output price and the price of an input factor remain constant (say $\Delta p = \Delta w = 0$), then

$$\Delta r \Delta K \leq 0.$$

- Changes in the price of an input factor should have the opposite sign of changes in the corresponding demanded factor quantities.

3.9. Current Field Developments

- Modern Dynamic Stochastic General Equilibrium (DSGE) macroeconomic models used by central banks have extensive micro-founded industries with profit maximizing firms under different competition structures.
- Market studies use microeconomic models with profit maximizing firms under different competition structures to obtain estimates of market power.
- Other firm objectives are not very commonly used in applications.
- Past profits are one of the most prominent (but not the only one) measures of a firm's performance.

3.10. Concise Summary

- Profit maximization is ubiquitous in economic theory and modeling applications.
- Firms determine input (factor demand) and output levels to maximize profits.
- Very simple idea, but not always realistic.
- Managers divert from profit maximization to achieve other strategic goals.
- However, completely disregarding profit maximization for long periods can lead to business failures.

3.11. Further Reading

- Varian (2010, chap. 20)
- CORE Team (2017, secs. 7.3-7.5)
- Denning (2014)
- Gast et al. (2020)

3.12. Mathematical Details

3.12.1. Profits

The general profit function of a **m-output, n-input** production process is given by

$$\pi(q_1, \dots, q_m, x_1, \dots, x_n) = \underbrace{\sum_{i=1}^m p_i q_i}_{\text{Revenue}} - \underbrace{\sum_{i=1}^n w_i x_i}_{\text{Cost}}.$$

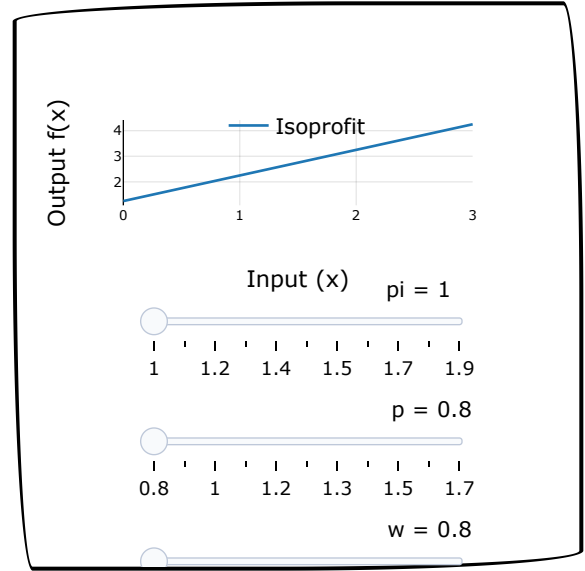
3.12.2. Isoprofit Loci

The isoprofit loci give the combinations of inputs and outputs resulting in a given level of profit $\hat{\pi}$, i.e.,

$$\left\{ (q, x) \in \mathbb{R}^m \times \mathbb{R}^n : \hat{\pi} = \sum_{i=1}^m p_i q_i - \sum_{i=1}^n w_i x_i \right\}.$$

For the single output, single input case ($m = 1 = n$), we obtain the isoprofit line in the output-input space by solving for q . Thus,

$$q = \frac{\hat{\pi}}{p} + \frac{w}{p} x.$$



3.12.3. Fixed and Variable Factors

If a factor is fixed, then there is a fixed cost component. E.g., say that x_j is fixed equal to $\hat{x}_j$. Then

$$\pi = \sum_{i=1}^n p_i q_i - \underbrace{\sum_{i \neq j} w_i x_i}_{\text{Variable cost}} - \underbrace{w_j \hat{x}_j}_{\text{fixed cost}}.$$

3.12.4. Single Factor Profit Maximization

The firm maximizes its profit by choosing the level of the input factor

$$\begin{aligned} & \max_x \{pq - wx\} \\ & \text{s.t. } q \leq f(x). \end{aligned}$$

Because profits are increasing in q , if the production function is monotonic (the usual case), the problem simplifies to

$$\max_x \{pf(x) - wx\}.$$

For non-boundary solutions, the marginal rates of revenue and cost are equalized at the maximum (why?), namely

$$pf'(x) = w.$$

1. Factor demand

How much from each factor would a profit maximizing firm like to use? The profit maximization optimality condition determines the factor demand as

$$x(w, p) = (f')^{-1}\left(\frac{w}{p}\right)$$

On some occasions, it is more convenient to avoid inverting f' and directly work with the optimality condition. This gives us the factor price as a function of the demanded factor quantity, so

$$w(x) = pf'(x).$$

We then say that we work with the inverse factor demand function.

3.12.5. Multiple Factor Profit Maximization

The firm maximizes its profit by choosing the levels of the input factors

$$\begin{aligned} \max_{x_1, x_2} \{pq - w_1x_1 - w_2x_2\} \\ \text{s.t. } q \leq f(x_1, x_2). \end{aligned}$$

Because profit is increasing in q , as long as the production function is monotonic (the usual case), the problem simplifies to

$$\max_{x_1, x_2} \{pf(x_1, x_2) - w_1x_1 - w_2x_2\}.$$

For non-boundary solutions, the marginal rates of revenue and cost are equalized for each factor at the maximum, namely

$$\begin{aligned} p \frac{\partial f(x_1, x_2)}{\partial x_1} &= w_1 \\ p \frac{\partial f(x_1, x_2)}{\partial x_2} &= w_2. \end{aligned}$$

Combining the first order conditions, we see that the marginal rate of technical substitution is equal to relative prices at any interior maximum, i.e.,

$$\text{MRTS}(x_1, x_2) = \frac{\partial f(x_1, x_2)/\partial x_1}{\partial f(x_1, x_2)/\partial x_2} = \frac{w_1}{w_2}.$$

3.12.6. Non-zero Profits are Incompatible with Constant Returns to Scales

Suppose that (x_1, x_2) maximizes profits under a constant returns to scales production technology. Then, for any $t > 1$,

$$\begin{aligned} \pi(tx_1, tx_2) &= pf(tx_1, tx_2) - w_1tx_1 - w_2tx_2 \\ &= t(pf(x_1, x_2) - w_1x_1 - w_2x_2) \\ &= t\pi(x_1, x_2). \end{aligned}$$

If $\pi(x_1, x_2)$ is positive, then $t\pi(x_1, x_2) > \pi(x_1, x_2)$. Since (x_1, x_2) is profit maximizing, $\pi(x_1, x_2)$ cannot be positive. Repeating the argument with $0 < t < 1$, we can exclude the possibility of negative profits. Therefore, only zero maximizing profits are compatible with constant returns to scales.

3.13. Exercises

3.13.1. Group A

- Suppose that a firm produces output q using only labor L according to the production function $q = \sqrt{L}$. The price of output is $p = 2$, and the price of labor is $w = 1$. Find the profit maximizing labor and the corresponding output. What is the optimal profit?
- Suppose that a firm produces output q using labor L and capital K according to the production function $q = K^{1/4}L^{1/4}$. The price of output is $p = 2$, the wage is $w = 1$, and the interest rate is $r = 1$.
 - What happens to output when capital and labor are doubled?
 - Find the profit maximizing labor, capital, and output.
 - Find the maximum profits.
 - Is there a unique maximizing output if the production is given by $q = K^{1/2}L^{1/2}$? Why, or why not?

3.13.2. Group B

- Consider a firm with a two dimensional production function f .
 - What happens to the firm's profits if f exhibits increasing returns to scale and the firm doubles its scale of operation while all prices remain fixed?
 - What happens to the firm's profits if f exhibits decreasing returns to scale and the firm halves its scale of operation while all prices remain fixed?
- Consider a firm with a single input factor production technology f having diminishing marginal product. The output and input prices, p and w , are fixed. The firm produces at a positive output level for which $pf'(x) > w$ holds and makes positive profits. Is this a profit maximizing firm? If not, and the firm wishes to increase its profits, should it increase or decrease the amount of the input factor?
- Consider a firm with a production function $f(x_1, x_2) = x_1^{1/2}x_2^{1/4}$. The output price is p , and the input prices are w_1 and w_2 . All prices are fixed.
 - Show that, when maximizing profits, the marginal product of each factor is equal to the corresponding real factor price.
 - Use your answer in the previous part to calculate the input factor demands.
 - Suppose that $p = 4$, $w_1 = 2$, and $w_2 = 1$. What are the optimal demanded quantities of the input factors? What is the optimal supplied quantity? How much profit does the firm make?
- Suppose that at the beginning of a given month, we observe that a firm produces an output level $q = 20$ at a price level $p = 2$. After a month, we observe that the same firm produces $q = 15$ at a price of $p = 4$. No other changes are observed in the prices of the market. Does the firm maximize its profit?

3.13.3. Group C

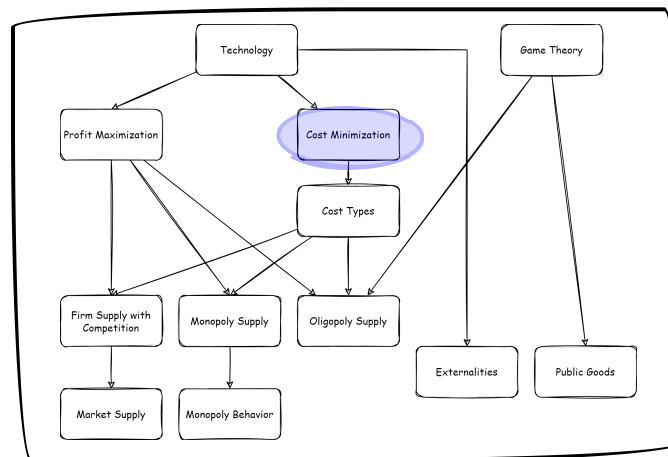
1. Consider a firm with a Cobb-Douglas production function $f(x_1, x_2) = Ax_1^\alpha x_2^{(1-\alpha)r}$ for $0 < \alpha < 1$ and $r \neq 1$. Output and input prices, namely p , w_1 , and w_2 , are fixed.
 - a. Write the profit maximization problem of the firm.
 - b. Calculate the first order conditions of the profit maximization problem.
 - c. Calculate the supply function.
 - d. Calculate the input factor demands.
 - e. Calculate the profit of the firm.
 - f. What happens to supply, input factor demands, and profit for $r = 1$?

4. Cost Minimization

4.1. Context

- In contrast to profit maximization, cost minimization is less controversial. Economists and managers agree that minimizing production costs is good practice.
- In fact, cost minimization arguments frequently appear in political discussions about minimum wages.
 - How is cost minimization different from profit maximization?
 - How is it similar to profit maximization?
 - How is it used in political debates?

4.2. Course Structure Overview



4.3. Lecture Structure and Learning Objectives

Structure

- Minimum Wage in Germany (Case Study)
- Basic Concepts
- Examples
- Analysis of the Minimum Wage Argument and its Limitations
- Current Field Developments

Learning Objectives

- Explain why cost maximization has such a prominent role in modern applications.
- Describe the solution of the cost minimization problem with a single input.
- Describe the solution of the cost minimization with multiple inputs.
- Analyze the relationship between conditionally demanded input factors and prices.
- Apply the concept of cost minimization to analyze the minimum wage policy.

4.4. Minimum Wage in Germany

- The minimum wage (Mindestlohn) law was introduced on January 1st, 2015.
- Its introduction was one of the most controversial topics during the 2013 elections.
- The SPD, the Greens, and the Left party were in favor. The FDP and CDU were less keen.

4.4.1. Why are Minimum Wage Laws So Controversial?

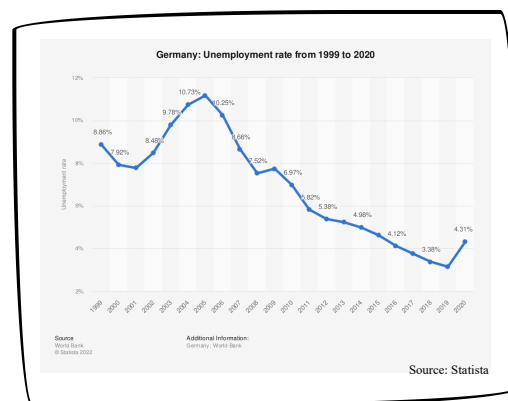
- The minimum wage increases the labor cost of production →
- Firms have an incentive to substitute labor with other input factors (e.g., capital) while keeping production constant (based on the MRTS) →
- This can decrease labor demand (conditional factor demand for labor) →
- Lead to job losses and unemployment.

4.4.2. Pessimistic Predictions

- The Ifo Institute of Munich was painting a horror scenario at the time (2014).
- It calculated that a minimum wage of **8.50** Euros would jeopardize **0.9** million jobs!
- Its predictions in 2010 were even worse, with **1.2** million jobs on the line!
- The reports of Ifo cannot be found online anymore.
- But you can follow the story by Krüsemann ([2014](#))

4.4.3. A Less Harsh Reality

- The reality was much less catastrophic.
- Evidence suggests that the policy did not lead to job losses.
- In fact, they suggest that the policy led to some reduction in inequalities across individuals and regions ([Ahlfeldt, Roth, and Seidel 2018](#)).



4.4.4. Why some Predictions Were So Off?

- The cost minimization channel is sensible!
- However, it is not the only channel affecting labor demand.
- Migration affects the number of available workers and changes the labor supply.
- Changes in demand for exports affect labor demand.
- What happens in the market is a combination of demand and supply.
- These effects may reinforce or cancel out each other.

4.5. Cost Minimization

- **Cost minimization** is a process in economic modeling and business practice by which firms choose the input factors producing a given level of output at the lowest cost.

4.5.1. Production Expenditures

- The total production expenditures for using capital K and labor L are calculated by

$$E(K, L) = rK + wL$$

- Expenditures are measured in monetary units per time (e.g., Euros per week, Dollars per month, etc.).
- They do not directly depend on production technology.
- They do not directly depend on the produced quantity.
- They are the same if we use $K = 2$ and $L = 1$ to produce $q = 1$ or $q = 2$.
- Not very helpful.
- How can we find the minimum cost of producing a given quantity?

4.5.2. Example: Cost Minimization with a Root Production Function

- Suppose production is only based on labor, i.e.,

$$q = \sqrt{L}.$$

- And that we want to find the minimum cost of producing $q_0 = 2$ when the wage is $w = 1$.
- This means that $L = 4$.
- So, the minimum cost is $wL = 4$.

4.6. Isocost Lines

- When there are many input factors, many different combinations of inputs can result in the same cost.
- An **isocost** curve is a locus of points showing all the input combinations resulting in equal production costs.
- For example, we fix $\hat{c} = E(K, L)$ and solve for capital to get

$$K = \frac{1}{r}\hat{c} - \frac{w}{r}L$$

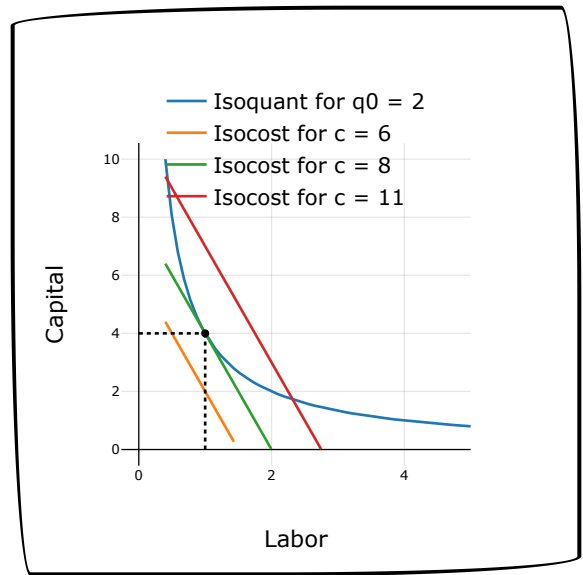
4.6.1. Example: Cost Minimization with a Cobb-Douglas Production Function

- Suppose production is given by

$$q = K^{1/2}L^{1/2}.$$

- We want to find the minimum cost of producing $q_0 = 2$
- For the cost minimizing allocation, the MRTS is equal to the ratio of factor prices (slope of the isocost line).

$$\text{MRTS}(K, L) = \frac{\partial f(K, L)/\partial K}{\partial f(K, L)/\partial L} = \frac{r}{w}$$



4.7. Conditional Factor Demand

- Substituting the partial derivatives in the optimality condition gives

$$\frac{L}{K} = \frac{r}{w} \implies L = \frac{r}{w}K$$

- Substituting in the production function yields

$$q = \left(\frac{r}{w}\right)^{1/2} K \implies K = \left(\frac{w}{r}\right)^{1/2} q$$

- Similarly, we obtain (Exercise)

$$L = \left(\frac{r}{w}\right)^{1/2} q$$

- The **conditional factor demand** is a function that gives the cost minimizing input factor quantity for a given output quantity and input prices.

4.8. Cost Function

- Using the conditional factor demands, we can calculate the minimum expenditure required for producing q

$$c(q) = r\left(\frac{w}{r}\right)^{1/2} q + w\left(\frac{r}{w}\right)^{1/2} q = 2(rw)^{1/2} q$$

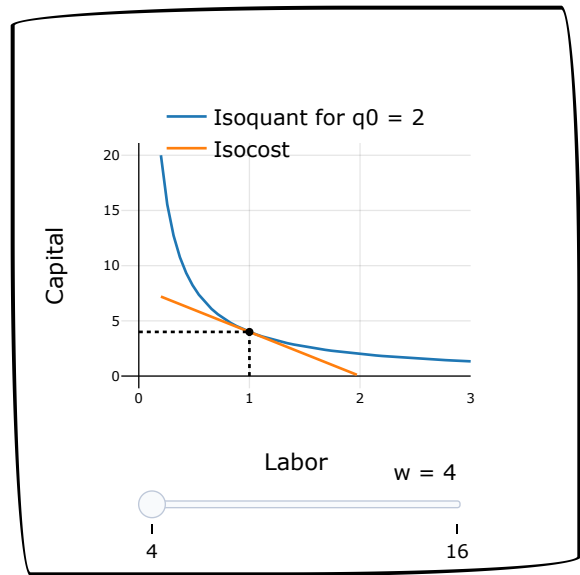
- The **cost function** gives the minimum cost required for producing an output quantity given input prices.

4.9. Application: What Happens when Wages Increase?

- Consider the Cobb-Douglas case and suppose that wages increase ($w \uparrow$).
- Conditional demand for labor decreases ($L = \left(\frac{r}{w}\right)^{1/2} q \downarrow$)
- Conditional demand for capital increases ($K = \left(\frac{w}{r}\right)^{1/2} q \uparrow$)
- Production cost increases ($c(q) = 2(rw)^{1/2} q \uparrow$)

4.9.1. Substitution Effect

- The slope of the isocost line (relative prices) changes.
- Firms substitute the newly more expensive labor with capital to maintain production constant.



4.10. Profit Maximization vs Cost Minimization

	Profit Maximization	Cost Minimization
Objective	Maximize profits	Minimize costs
Controls	Input factors	Input factors
Parameters	Input prices, output price	Input prices, output quantity
Optimal Control	Factor demand	Conditional factor demand
Value function	Profit function	Cost function

4.11. Special Type of Costs

4.11.1. Fixed Costs

- Sometimes, production involves costs that cannot be adjusted (at least in the short-run).
- For example, the offices rented by a firm cannot change on short notice.
- Then, one component of the cost function is fixed, i.e.,

$$c(q) = \alpha + \beta q$$

- **Fixed costs** are business costs that do not depend on the level of production. They are the same for every amount of produced output.

4.11.2. Sunk Costs

- Fixed costs may either be recoverable or not.
 - Suppose that a firm owns the property where its offices are located. If production ceases at some point, it can liquidate the property and recover some of its costs.
 - Suppose that a firm rents the property where its offices are located. If production ceases at some point, it cannot recover the rent that it paid during its operation.
- **Sunk costs** are fixed business costs that cannot be recovered if production stops.
- Bygones are bygones: Sunk costs do not affect the future choices of (rational decision making) firms.

4.11.3. Average Cost Function

- In business practice, it is more useful to talk about the average production cost.
- The **average cost** is the mean production cost of a cost minimizing firm producing an output quantity under given input prices.

$$\bar{c}(q) = \frac{\text{Total cost}}{\# \text{Produced units}} = \frac{c(q)}{q}$$

4.12. Revealed Cost Minimization Inequality

- Suppose that we observe the choices (K_1, L_1) and (K_2, L_2) of a cost minimizing firm on dates **1** and **2**.
- Since (K_1, L_1) is cost minimizing under prices (r_1, w_1) , it should hold

$$r_1 K_1 + w_1 L_1 \leq r_1 K_2 + w_1 L_2.$$

- Since (K_2, L_2) is cost minimizing under prices (r_2, w_2) , it should hold

$$r_2 K_2 + w_2 L_2 \leq r_2 K_1 + w_2 L_1.$$

- Adding these two inequalities gives

$$\Delta r \Delta K + \Delta w \Delta L \leq 0.$$

- How can we interpret this inequality?

4.13. Current Field Developments

- Cost minimization processes are often used in applications.
- In business, cost minimization is often used by firms because tracking (marginal) costs is easier than estimating marginal revenues (which is required for profit maximization).
- Other cost minimization applications focus on
 - Inventory management problems
 - Transportation problems (with applications going beyond economics)
 - Datacenter (i.e., data storage) management

4.14. Concise Summary

- Cost minimization is a process with many useful applications in economic modeling and business practice.
- Firms determine the inputs minimizing their costs conditional on producing a given output level.
- Cost minimization does not require information on marginal revenue (because the output level is given).
- In principle, it is easier for firms to follow because less information is required compared to profit maximization.
- A very simple idea with many practical applications.
- However, oversimplifications should be avoided as they can lead to inaccurate predictions.

4.15. Further Reading

- Varian ([2010, chap. 21](#))
- CORE Team ([2017, sec. 7.3](#))
- Krüsemann ([2014](#))
- Ahlfeldt, Roth, and Seidel ([2018](#))

4.16. Mathematical Details

4.16.1. Production Expenditures

A firm employing input factors $(x_1, \dots, x_n)$ under prices $(w_1, \dots, w_n)$ has expenditures

$$E(x_1, \dots, x_n) = \sum_{j=1}^n x_j w_j$$

If a factor is fixed, then there is a fixed component to expenditures. For instance if x_1 is fixed to $\hat{x}_1$, we have

$$c(x_2, \dots, x_n) := c(\hat{x}_1, x_2, \dots, x_n) = \underbrace{\hat{x}_1 w_1}_{\text{fixed cost, say } w_0} + \sum_{j=2}^n x_j w_j$$

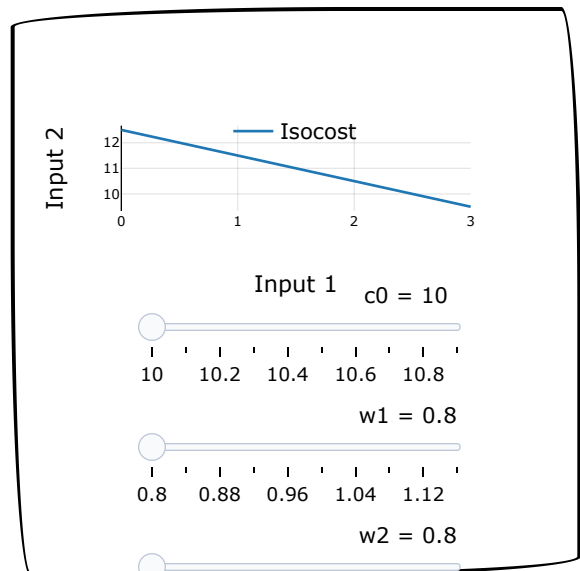
4.16.2. Isocost Lines

The isocost loci give the combinations of inputs resulting in a given level of costs $\hat{c}$, i.e.,

$$\left\{ x \in \mathbb{R}^n : \hat{c} = \sum_{j=1}^n w_j x_j \right\}.$$

For the case of two inputs ($n = 2$), we obtain the isocost line in the input plane by solving for x_2 . Thus,

$$x_2 = \frac{\hat{c}}{w_2} - x_1 \frac{w_1}{w_2}$$



4.16.3. Single Factor Cost Minimization

The problem statement is

$$\begin{aligned} & \min_x \{w_0 + wx\} \\ \text{s. t. } & f(x) \geq q. \end{aligned}$$

If the production function is (strictly) monotonic (for instance, a root production function) and the factor price w is positive, then the solution to the problem is obtained by merely inverting the production function. Thus,

$$\begin{aligned} x(q) &= f^{-1}(q), \\ c(q, w) &= w_0 + wx(q). \end{aligned}$$

4.16.4. Multiple Factor Cost Minimization

The problem is given by

$$\begin{aligned} & \min_{x_1, x_2} \{w_1 x_1 + w_2 x_2\} \\ \text{s. t. } & f(x_1, x_2) \geq q. \end{aligned}$$

For non-boundary solutions, a variations arguments shows that the ratio of marginal products is equal to the ratio of factor prices at interior, cost minimizing allocations. This means

$$\text{MRTS}(x_1, x_2) = \frac{\partial f(x_1, x_2)/\partial x_1}{\partial f(x_1, x_2)/\partial x_2} = \frac{w_1}{w_2}.$$

1. Conditional Factor Demand

The last equation typically results in a relationship between the two input factors x_1 and x_2 . One can usually eliminate one of the two factors and express the remaining factor as a function of the produced quantity by combining this relationship with the problem's constraint (the production function inequality). This determines the conditional (on the output level) factor demands

$$x_i(q, w_1, w_2) \quad (i = 1, 2),$$

which show how much from each factor would a cost minimizing firm aiming to produce q under prices q would like to hire. When the discussion focuses on the relationship of demanded input factors with output, prices are typically omitted from the argument list, and we simply write

$$x_i(q) \quad (i = 1, 2).$$

2. Cost function

The cost function can be obtained by

$$c(q, w_1, w_2) = w_1 x_1(q, w_1, w_2) + w_2 x_2(q, w_1, w_2).$$

Similarly to the conditional factor demand, prices are often omitted from the cost function's arguments, i.e.,

$$c(q) = w_1 x_1(q) + w_2 x_2(q)$$

4.16.5. Implications of Returns to Scales on Costs

If the production technology exhibits constant returns to scale, then

$$c(tq) = tc(q) \quad (t > 0).$$

If the production technology exhibits increasing returns to scale, then

$$c(tq) < tc(q) \quad (t > 1).$$

Lastly, for production technologies exhibiting decreasing returns to scale, we have

$$c(tq) > tc(q) \quad (t > 1).$$

4.17. Exercises

4.17.1. Group A

- Consider a firm with a production function $f(x_1, x_2) = 4x_1^{1/2} x_2^{1/2}$. The input prices are $w_1 = 40$ and $w_2 = 10$.
 - What is the slope of the isocost lines with respect to the first input factor?
 - What is the cost minimizing marginal rate of technical substitution between x_1 and x_2 ?
 - Find the cost minimizing conditional factor demanded quantities and the minimum production cost when the firm produces an output level q .
 - Calculate the cost minimizing conditional factor demanded quantities and the minimum production cost when $q = 40$.
- Consider a firm with a production technology f having two factors that exhibit diminishing marginal rates of technical substitution. The input factor prices, w_1 and w_2 , are fixed. The firm produces an output level q using input factors quantities such that $\text{MP}(x_1; x_2)/w_1 > \text{MP}(x_2; x_1)/w_2$ holds. Is this a cost minimizing firm? If not, how can the firm reduce its cost?
- Suppose that at the beginning of a given month, we observe that a firm produces an output level q using input factors $x_1 = 1$ and $x_2 = 3$ with corresponding prices $w_1 = 3$ and $w_2 = 1$. After a month, we observe that the same firm produces the same level of output using $x_1 = 5$ and $x_2 = 1$ under prices $w_1 = 5$ and $w_2 = 3$. Does the firm minimize its cost?
- Consider a firm with a Cobb-Douglas production function $f(x_1, x_2) = 2x_1^{\frac{1}{3}} x_2^{\frac{1}{3}}$. Input prices, namely w_1 , and w_2 , are fixed.

- What returns to scale does the production function have?
- Write the Lagrangian of the cost minimization problem of the firm.
- Calculate the first order conditions of the cost minimization problem.
- Calculate the conditional input factor demands.
- Calculate the cost function of the firm.

4.17.2. Group B

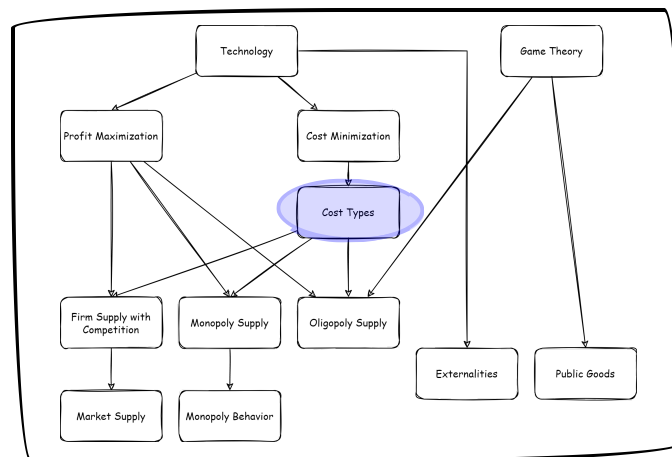
- Prove that if a firm maximizes its profits, then it minimizes its costs.
- What are the differences between the conditional factor demand and the factor demand? How are they related?
- Consider a firm with a Cobb-Douglas production function $f(x_1, x_2) = Ax_1^\alpha x_2^{(1-\alpha)r}$ for $0 < \alpha < 1$ and $r \neq 1$. Input prices, namely w_1 , and w_2 , are fixed.
 - Write the Lagrangian of the cost minimization problem of the firm.
 - Calculate the first order conditions of the cost minimization problem.
 - Calculate the conditional input factor demands.
 - Calculate the cost function of the firm.

5. Cost Types

5.1. Context

- Not all costs have the same characteristics. Economists talk about fixed and variable costs and distinguish between short- and long-run horizons.
- Different costs sway (rational) business decisions in distinct ways.
 - How do fixed and variable costs differ?
 - How do short- and long-run costs differ?
 - How do these differences affect business decisions?

5.2. Course Structure Overview



5.3. Lecture Structure and Learning Objectives

Structure

- The Sunk Cost Fallacy (Case Study)
- Basic Concepts
- Examples
- A Practical Exercise
- Current Field Developments

Learning Objectives

- Explain the various types of economic costs.
- Describe the relationships between average, marginal, fixed, and total costs.
- Explain the concept of short-run costs and its relation to fixed costs.
- Describe the relationships between short- and long-run costs.
- Apply the new concepts to a practical exercise.

5.4. The Sunk Cost Fallacy

- Imagine that you and your friend have booked a wake-boarding weekend in Waldsee for **100** Euros each.
- Your friend approaches you after a couple of weeks and suggests you go for wake-boarding in Lautenbourg.
- You happily agree, and you book for **50** Euros each.
- You realize later that both bookings were for the same weekend!
- Although you like Waldsee a lot, you are sure that you will enjoy more a weekend in Lautenbourg.
- Where would you eventually choose to go?

5.4.1. Irrational Choices

- Many behavioral economists in the 1980s contested that humans always make rational choices, as mainstream economic theory suggests.
- Among the examples of irrational behavior was the Sunk Cost Fallacy.
- Once time, money, or effort is invested in an act or a choice, humans tend to maintain their choices.
- Economists started conducting experiments to examine if the last conjecture was true.

5.4.2. The Ski Trip

Assume that you have spent **\$100** on a ticket for a weekend ski trip to Michigan. Several weeks later you buy a **\$50** ticket for a weekend ski trip to Wisconsin. You think you will enjoy the Wisconsin ski trip more than the Michigan ski trip. As you are putting your just-purchased Wisconsin ski trip ticket in your wallet, you notice that the Michigan ski trip and the Wisconsin ski trip are for the same weekend! It's too late to sell either ticket, and you cannot return either one. You must use one ticket and not the other. Which ski trip will you go on?

([Arkes and Blumer 1985](#) Experiment 1)

Choices	Nobs	Sample%
\$100 ski trip to Michigan	33	54.10
\$50 ski trip to Wisconsin	28	45.90
Total	61	

5.4.3. Can This Happen in Real Life Too?

- Offered discounted seasonal theater tickets to subjects and checked how often people attended. ([Arkes and Blumer 1985](#) Experiment 2)
- The first **60** people approaching the ticket window to buy a ticket were randomly split into three groups:
 - Those who paid the normal **\$15** price,
 - those who took **\$2** discount, and
 - those who took a **\$7** discount.
- Tracked the number of times each individual visited the theater.

Group	Average No. Visits
No discount	4.11
\$2 discount	3.32
\$7 discount	3.29

5.4.4. Rational Behavior

- Mainstream economic theory suggests that rational agents should not base their decisions on sunk costs.
- Only future benefits and costs determine the optimality of decisions, as those in the past cannot change anymore.
- Acting "irrationally" at a personal level is one thing.
- As a manager, falling to sunk cost fallacies can cost money and endanger your business!

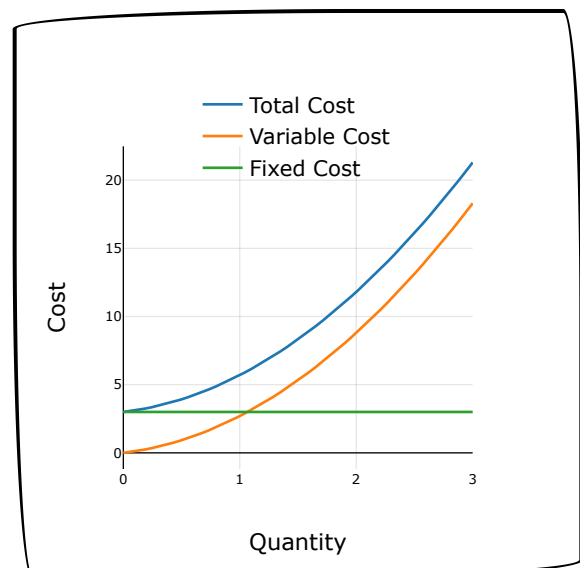
5.5. Total, Variable, and Fixed Cost

- The cost function can be decomposed into two parts.
- A fixed cost component that does not change with the level of production output.
- A **variable cost** component that depends on the level of produced output.

$$\text{Total Cost} = \text{Variable Cost} + \text{Fixed Cost}$$

$$c(q) = \mu(q) + \sigma(q)$$

- Geometrically, the total cost is a vertical, upward shift of variable cost by the amount of fixed cost.



5.6. A Practical Exercise

Case	Cost Type
Real estate rents	Fixed
Direct materials used for a product	Variable
Salaries (fixed employee compensation)	Fixed
Piece Rate (variable employee compensation, e.g., bonuses)	Variable
Production supplies (e.g., machine oil)	Variable
Utilities (e.g., electricity, telecommunication)	Fixed/Variable
Property taxes	Fixed
Sale taxes	Variable
Insurance	Fixed
Depreciation	Fixed
Shipping costs	Variable

5.7. Average Total, Variable, and Fixed Cost

- Similarly, average costs can be decomposed into two parts.
- Average fixed cost** is the component of the total cost that does not change with the production level (i.e., fixed cost) per unit of produced output.
- Average variable cost** is the component of the total cost that changes with the production level (i.e., variable cost) per unit of produced output.

$$\begin{array}{l} \text{Average} \quad \text{Average} \quad \text{Average} \\ \text{Total} \quad = \text{Variable} + \text{Fixed} \\ \text{Cost} \quad \quad \text{Cost} \quad \quad \text{Cost} \end{array}$$

$$\begin{aligned} \bar{c}(q) &= \frac{c(q)}{q} \\ &= \frac{\mu(q)}{q} + \frac{\sigma(q)}{q} \\ &= \bar{\mu}(q) + \bar{\sigma}(q) \end{aligned}$$

5.8. Marginal Cost

- The **marginal cost** is the rate of change of the total production cost with respect to the output level. It shows how much total cost increases when the output is marginally increased. It is given by the cost function's derivative, i.e., c' .
- Similarly, the **marginal variable cost** is the rate of change of the variable cost with respect to the output level. It shows how much the variable cost increases when the output is marginally increased. It is given by the derivative of the variable cost function, i.e., μ' .
- Analogously, one can define the **marginal fixed cost** as the rate of change of the fixed cost with respect to the output level. Since fixed cost is constant, the marginal fixed cost is always zero.

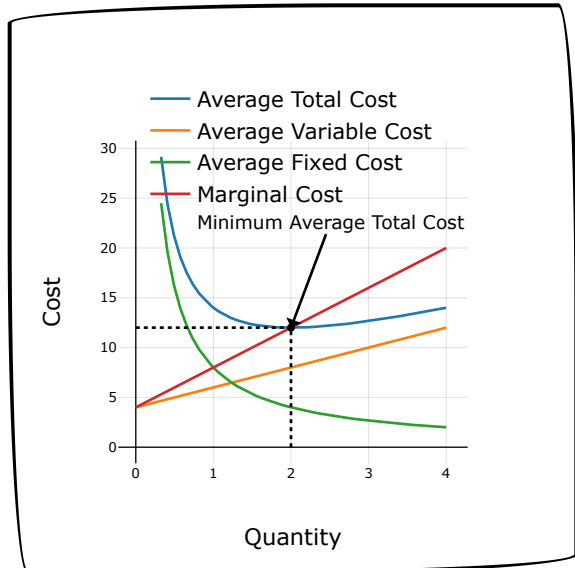
5.9. Example: Calculating Costs

- Calculate the costs:

Cost type	Cost expression
Total cost	$c(q) = 3 + 2q^2$
Variable Cost	$\mu(q) = 2q^2$
Fixed Cost	$\sigma(q) = 3$
Average Total Cost	$\bar{c}(q) = \frac{3}{q} + 2q$
Average Variable Cost	$\bar{\mu}(q) = 2q$
Average Fixed Cost	$\bar{\sigma}(q) = \frac{3}{q}$
Marginal Total Cost	$c'(q) = 4q$
Marginal Variable Cost	$\mu'(q) = 4q$
Marginal Fixed Cost	$\sigma'(q) = 0$

5.10. Minimizing Average Cost

- The average total cost is initially decreasing because it is dominated by the average fixed cost.
- Eventually, the average variable cost becomes the main driving force, and the average total cost becomes increasing.
- The minimum average cost (i.e., the most economical production per unit!) is attained when the marginal cost intersects the average total cost.

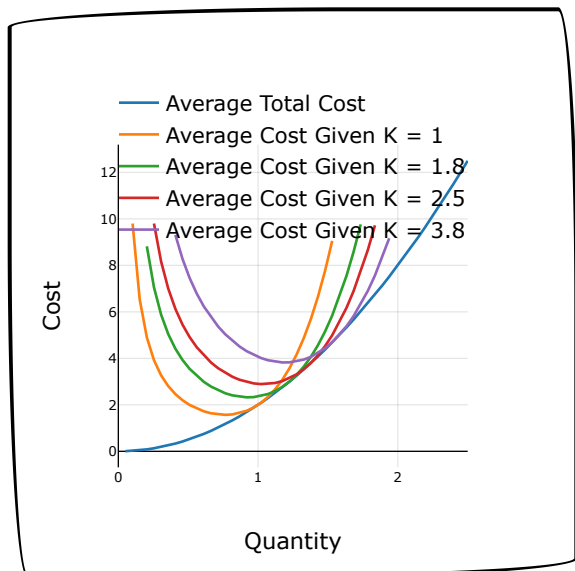


5.11. Short-Run and Long-Run Costs

- The firm might not be able to adjust all production factors in short periods of time.
- E.g., some assets and properties are not always liquid.
- Some costs can then be fixed in the short-run.
- Short-run costs are greater than long-run costs (why?).

5.12. Short-Run and Long-Run Cost Folding

- Suppose $f(K, L) = K^{\frac{1}{3}}L^{\frac{2}{3}}$, $r = w = 1$.
- Suppose that capital is fixed in the short-run, say $\hat{K}_0 = 1$ or $\hat{K}_1 = 8$.
- How can we depict the short-run and long-run relation?
- The short-run curve is tangent (from above - greater cost) to the long-run cost curve.



5.13. Current Field Developments

- Well-established concepts (the cost typology) with long-standing applications.
- In merger analysis and antitrust cases, the marginal cost estimation is a central issue with many new proposed approaches in recent years.
- Recently, the sunk cost fallacy was demonstrated for non-human animals too ([Sweis et al. 2018](#))

5.14. Concise Summary

- Different cost concepts are used in economics.
- Average costs explain production costs per unit.
- Marginal costs explain the production costs of small changes in the produced output.
- Average production costs are minimized when they are equal to marginal production costs.
- Short-run costs are greater than long-run costs because some factors might not be flexible in the short-run.
- Understanding the cost types a firm deals with is important in reaching rational business decisions.

5.15. Further Reading

- Varian (2010, chap. 22)
- CORE Team (2017, secs. 7.3, 7.3.1)
- Arkes and Blumer (1985)

5.16. Mathematical Details

Working with the cost function has the advantage that cost can be expressed in terms of a single variable (production output). Irrespective of the number of input factors involved in the cost minimization problem, the cost function (if it exists) involves only output as a variable. This is illustrated in the following two examples.

5.16.1. Cost Functions Induced by Root Production Functions

Consider the cost minimization problem with a root production function

$$\begin{aligned} \min_x \{w_0 + wx\} \\ \text{s.t. } q \leq f(x) = x^r \quad (0 < r < 1). \end{aligned}$$

Its solution, namely the (total) cost function, is given by

$$c(q) = w_0 + wq^{\frac{1}{r}}.$$

Having the cost function, one can calculate the remaining cost types as in the following table

Cost type	Cost expression
Total cost	$c(q) = w_0 + wq^{\frac{1}{r}}$
Variable cost	$\mu(q) = wq^{\frac{1}{r}}$
Fixed cost	$\sigma(q) = w_0$
Average cost	$\bar{c}(q) = \frac{w_0}{q} + wq^{\frac{1-r}{r}}$
Average variable cost	$\bar{\mu}(q) = wq^{\frac{1-r}{r}}$
Average fixed cost	$\bar{\sigma}(q) = \frac{w_0}{q}$
Marginal cost	$c_q(q) = \frac{w}{r} q^{\frac{1-r}{r}}$
Marginal variable cost	$\mu_q(q) = \frac{w}{r} q^{\frac{1-r}{r}}$
Marginal fixed cost	$\sigma_q = 0$

5.16.2. Cost Functions Induced by Cobb-Douglas Production Functions

The cost minimization problem for a Cobb-Douglas production function is

$$\begin{aligned} \min_{x_1, x_2} \{w_1 x_1 + w_2 x_2\} \\ \text{s.t. } q \leq f(x) = A(x_1^\alpha x_2^{1-\alpha})^r \quad (0 < \alpha < 1, 0 < r < 1), \end{aligned}$$

and its solution is given by

$$c(q) = \left(\frac{q}{A}\right)^{\frac{1}{r}} w_1^\alpha w_2^{1-\alpha} \left(\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \right).$$

5.16.3. Minimizing Average Cost

The minimum average cost is attained at the output level for which the marginal cost equals the average cost. Namely

$$\min_q \bar{c}(q) = c'(\arg \min_q \bar{c}(q))$$

If c is differentiable, we can show this result by setting the derivative of average cost equal to zero, i.e.,

$$\begin{aligned} \bar{c}'(q) = \frac{c'(q)q - c(q)}{q^2} = \frac{c'(q) - \mu(q)}{q} \stackrel{!}{=} 0 \implies \\ c'(q) \stackrel{!}{=} \mu(q) \end{aligned}$$

Analogously, we can show that the minimum average variable cost is attained at the output level for which the marginal cost equals the average variable cost.

$$\min_q \bar{\mu}(q) = c'(\arg \min_q \bar{\mu}(q)).$$

5.16.4. The Short- and Long-Run Relationship

Consider again the cost minimization problem

$$\begin{aligned} & \min_{x_1, x_2} \{w_1 x_1 + w_2 x_2\} \\ \text{s. t. } & q \leq f(x) = A(x_1^\alpha x_2^{1-\alpha})^r \quad (0 < \alpha < 1, 0 < r < 1), \end{aligned}$$

with resulting cost function

$$c(q) = \left(\frac{q}{A}\right)^{\frac{1}{r}} w_1^\alpha w_2^{1-\alpha} \left(\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} + \left(\frac{\alpha}{1-\alpha}\right)^{-\alpha} \right).$$

What happens if one production function, say x_2 , is fixed? It is common for many production settings that some factors cannot be flexibly adjusted in short periods. This results in a fixed cost component. Fixing the level of $x_2 = \hat{x}_2$, the minimization problem becomes

$$\begin{aligned} & \min_{x_1} \left\{ w_1 x_1 + \underbrace{w_2 \hat{x}_2}_{w_0} \right\}, \\ \text{s. t. } & q = f(x_1) = \underbrace{\left(A \hat{x}_2^{(1-\alpha)r} \right)}_B x_1^{\frac{\beta}{\alpha r}} \quad (0 < \alpha < 1, 0 < r < 1). \end{aligned}$$

Its solution is given by

$$c(q; \hat{x}_2) = \left(\frac{q}{A}\right)^{\frac{1}{\alpha r}} w_1 \hat{x}_2^{\frac{\alpha-1}{\alpha}} + w_2 \hat{x}_2.$$

Using the β, B notation, we observe that the production function essentially becomes a root function ($f(x_1) = Bx_1^{\frac{\beta}{\alpha}}$), and the resulting cost function is very similar to the cost function obtained by single variable, root production functions.

1. Cost folding

When all factors are allowed to vary, the cost is lower or equal than when at least one factor is fixed. Since x_2 is chosen to minimize costs if it is allowed to vary, it must hold

$$c(q) \leq c(q; \hat{x}_2).$$

Equality is attained when x_2 is fixed to the optimal conditional demanded quantity given the output level q , i.e.

$$c(q) = c(q; x_2(q)).$$

5.17. Exercises

5.17.1. Group A

1. Consider the cost function $c(q) = 16 + 4q^2$. Calculate and plot

- the average cost function,
- the marginal cost function,
- the average variable cost function.
- the average fixed cost function.
- What is the level of output that yields the minimum average production cost?

2. Consider the cost function obtained by a Cobb-Douglas production function

$$c(q; w_1, w_2) = \left(\frac{\alpha}{w_1}\right)^{-\alpha} \left(\frac{1-\alpha}{w_2}\right)^{-(1-\alpha)} \left(\frac{q}{A}\right)^{\frac{1}{r}},$$

and let $w_1 = w_2 = \alpha = 1/2$, $A = 1$, and $r > 0$. Show that the average cost function is

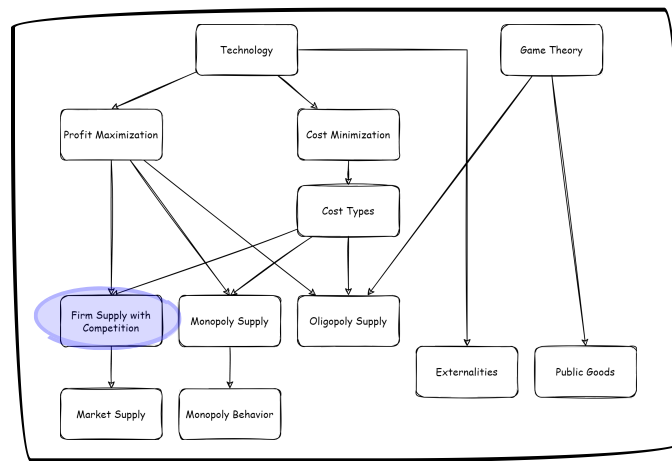
- increasing for decreasing returns to scale ($r < 1$),
- decreasing for increasing returns to scale ($r > 1$), and
- constant for constant returns to scale ($r = 1$).

6. Firm Supply

6.1. Context

- Competition is a frequent topic in political and economic discussion. Competition can constrain firms to act as price takers.
- Markets are not perfectly competitive, and firms are typically not purely price takers in reality. Nevertheless, such arguments remain central in economics, finance, and business.
 - Why is price taking behavior still so relevant?
 - How does competition restrain the behavior of firms?
 - How do firms decide how much to produce in competitive markets?

6.2. Course Structure Overview



6.3. Lecture Structure and Learning Objectives

Structure

- Microsoft's Pricing Strategies (Case Study)
- Basic Concepts
- Price Taking Examples
- Profit Maximization under Price Taking Exercise
- Current Field Developments

Learning Objectives

- Illustrate the implications of competition on individual firm supply.
- Explain how price taking firms decide how much to produce.
- Explain the shutdown decisions of competitive firms.
- Describe and compare the concepts of profit and producer's surplus.

6.4. Microsoft's Pricing Strategies

- In the early 1980s, several companies were competing in the operating system market of IBM-compatible PCs.
- In the 1990s and 2010s, Microsoft dominated the operating system market.
- In 2020s Microsoft's dominance stopped, and its operating system is nowadays the second most used.
- How did Microsoft manage to dominate the operating system market?
- How did it lose its primacy?

6.4.1. MS-DOS

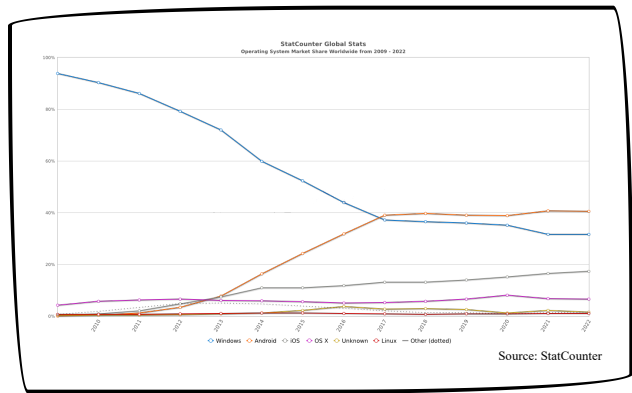
- In the early 1980s, the common practice of operating system companies was to charge hardware manufacturers for each operating system copy installed in a computer.
- Microsoft offered an alternative plan.
 - Charge computer manufacturers based on (the past number of) built computers.
 - The manufacturer was paying a general licensing fee and then could install the operating system in all the computers it produced.
 - Microsoft was offering low-priced licensing contracts making their operating system (MS-DOS) very attractive to manufacturers.

6.4.2. The Impact of Microsoft's Early Pricing Strategy

- Effectively, manufacturers could purchase Microsoft's operating system at much lower prices than the operating systems of other software companies.
 - A manufacturer had to pay **\$50 – \$100** for installing an alternative operating system on an additional machine.
 - It cost nothing to install MS-DOS on an additional machine once a licensing contract with Microsoft has been signed.
 - MS-DOS ended up being the default operating system

6.4.3. Android

- Android is a community (open source) operating system for mobile devices based on the Linux kernel.
- The wide use of smartphones and tablets drastically changed the operating system market.
- Although Microsoft offered an operating system suitable for smartphones and tablets, it did not manage to keep its primacy.



6.4.4. The Impact of Android on Microsoft's Pricing Strategy

- Android is free. Anyone can install the operating system on her device after accepting the terms and conditions.
- For mobile device manufacturers, Android is a cheaper operating system alternative for their products.
- This leads to more competitive prices for consumers too.
 - Microsoft's operating systems lost their primacy in the overall operating system market in **2017**.
 - Microsoft's operating systems are still dominant in less portable devices, such as desktop PCs and Laptops.
 - The rise of Android has also impacted Microsoft's pricing strategies for its desktop operating systems.
 - Licensed users were able to upgrade to the last two versions of Microsoft's operating system without paying for a new license.

6.5. Price Taking

- One prerequisite axiom of competitive market structure is price taking behavior.
- A firm (or a consumer) is a **price taker** if it cannot influence the market price on its own. Instead, price takers consider market prices as exogenous.

6.6. The Supply Function

- Supply is one of the two fundamental elements, with the other one being demand, determining the market state.
- The **firm's supply function** is a mapping that gives how much a profit maximizing firm would like to produce at a particular price.

6.6.1. The Shutdown Condition

- In the long-run, a firm operates if it makes non-negative profit.
- In the short-run, a firm might also keep producing while it makes losses.
- A firm shutdowns if the market price is below its average variable cost.
- Equivalently, a firm shutdowns if its revenue is not enough to cover, besides variable cost, at least some part of its fixed cost.

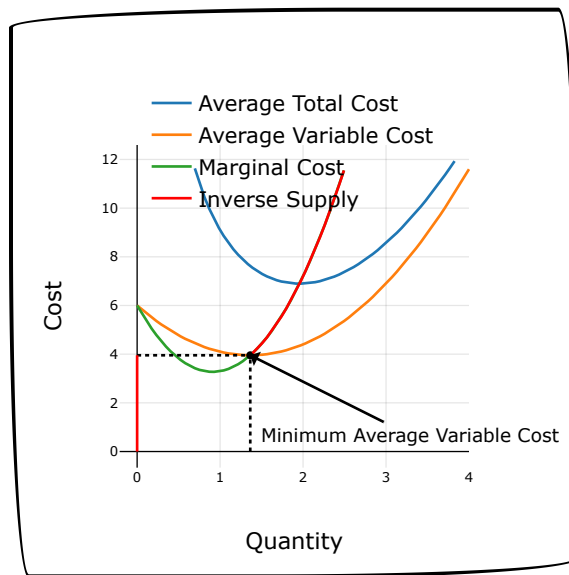
6.6.2. Price Taking Supply

- A price taking firm maximizes its profit when it produces at the point where its marginal cost is equal to the market price.
- For quantities where its marginal cost is above the market price,
 - the firm is making losses for the last unit of output it produces, and
 - it can increase its profit by lowering production.
- For quantities where its marginal cost is below the market price,
 - the firm is still making a profit for the last unit of output it produces, and
 - it can increase its profit even further by increasing production.

6.6.3. Inverse Supply

- The **firm's inverse supply function** is a mapping that gives at which price would a profit maximizing firm produce a particular quantity.
- It can be obtained by the first order condition of the firm's profit maximization problem.

6.6.4. Example: Price Taking Firm Supply

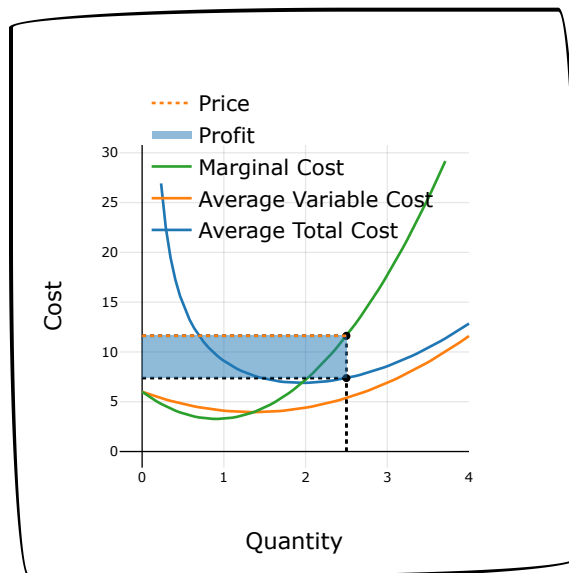


6.7. Economic Profit

- The (economic) **profit** or **loss** is the difference between the total revenue and the total cost of produced output.
- It is given by

$$\pi(q) = pq - c(q)$$

6.7.1. Example: Profit



6.7.2. Exercise: Profits of Price Taking Firms

- Suppose that the firm's cost function is $c(q) = 2q^2$.
- Suppose that the firm is a price taker, and the market price is $p = 8$.
- The firm maximizes its profit

$$\max_q \{8q - 2q^2\}.$$

- It solves the above to get

$$8 - 4q \stackrel{!}{=} 0 \implies q = 2.$$

- The firm makes profit

$$\pi = 16 - 8 = 8.$$

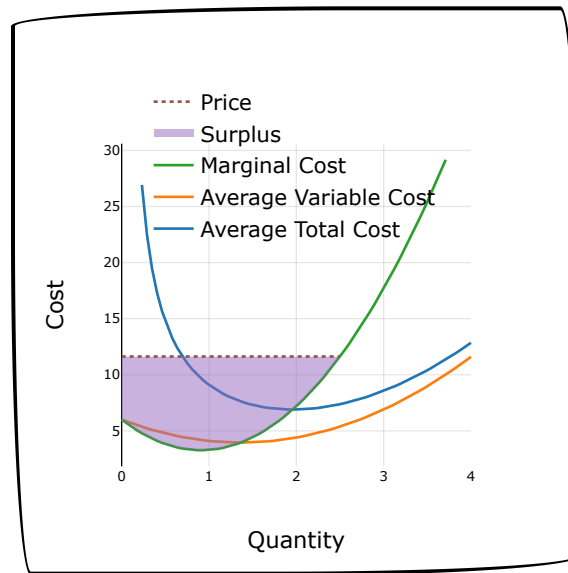
6.8. Producer's Surplus

- The **producer's surplus** is the accumulated amount that a producer benefits from selling each produced unit at a market price higher than its marginal cost (i.e., the least price the producer is willing to sell).
- It can be calculated by the cumulative difference between the marginal revenue and cost of each produced unit.

- It can also be calculated by the difference between total revenue and variable cost.

$$\Pi_s(q) = pq - \mu(q)$$

6.8.1. Example: Producer's Surplus



6.9. Producer's Surplus vs Profit

- If the production cost has a fixed component, a firm's profit differs from its producer's surplus by exactly this component.

$$\begin{aligned}\pi(q) &= pq - c(q) \\ \Pi_s(q) &= pq - \mu(q) \\ \Pi_s(q) - \pi(q) &= \sigma(q) = c(0) \geq 0\end{aligned}$$

6.10. Current Field Developments

- The majority of models in economics and finance examine firm supply under perfect competition terms.
- Industrial organization is a field of economics that uses market structures diverging from perfect competition, which can be more conducive.
- We will examine alternative market structures where firms make more strategic decisions in the subsequent topics.

6.11. Concise Summary

- Price taking behavior is not realistic, but it has predictive power in many competitive markets.
- Two lessons can be useful rules of thumb for business decisions in markets with intense competition:
 - Firms maximize their profits by choosing to produce so that the marginal cost equals the market price (marginal revenue).
 - They opt-out if the market price is below the average variable cost.
- The profit and producer's surplus measure the operational performance of firms.
- In the presence of fixed costs, profit and producer surplus are not identical.

6.12. Further Reading

- Varian ([2010, chap. 23](#))
- CORE Team ([2017, secs. 8.3, 8.8](#))

6.13. Mathematical Details

6.13.1. The Supply Function

The firm is willing to supply at prices covering its average variable cost. The cost function incorporates the technological constraints faced by the firm. Using the cost function, the profit maximization problem can be rewritten as

$$\max_q \{pq - c(q)\}.$$

The profit can be decomposed into two parts:

- revenue pq and
- cost $c(q)$.

For non-boundary solutions, the marginal revenue and marginal cost are equalized at the maximum (why?).

$$p = c'(q)$$

The above condition is necessary but not sufficient. The condition gives a maximum if

$$c''(q) > 0.$$

Therefore, marginal cost should be increasing (locally) at the maximum.

The firm opts out if for every level of output $q > 0$

$$\bar{\mu}(q) = \frac{\mu(q)}{q} = \frac{c(q) - c(0)}{q} \geq p.$$

The solution to the profit maximization problem determines the supply of the firm. Supply $s(p)$ gives the quantity that a profit maximizing firm would like to produce and sell in the market. On some occasions, it is easier to work with the inverse supply mapping $p(q)$, which gives the price for which the supplied quantity q is profit maximizing. The inverse supply can be directly obtained from the variational condition $p = c'(q)$.

6.13.2. A Cubic Cost Example

Consider the cubic cost function

$$c(q) = q^3 + c_2 q^2 + c_1 q + c_0.$$

The inverse supply of a profit maximizing firm is

$$p(q) = \begin{cases} [0, c_1 - \frac{c_2^2}{4}) & q = 0 \\ 3q^2 + 2c_2 q + c_1 & q \geq -\frac{c_2}{2} \end{cases}.$$

6.13.3. Producer's Surplus

The surplus of the producer is the cumulative difference between the marginal revenue and cost of every produced unit. In a single-price market (such as in the case of perfect competition), the revenue of each unit (marginal revenue) is equal to the market price. The cost per unit of production is the marginal cost. Then, we can calculate

$$\begin{aligned} \Pi_s(q) &= \int_0^q (p - c'(z)) dz \\ &= [pz - c(z)]_0^q \\ &= pq - (c(q) - c(0)) \\ &= pq - \mu(q) \end{aligned}$$

6.13.4. The Case of Constant Returns to Scale

If production exhibits constant returns to scale, then $c(q) = c_1 q$. Therefore, a profit maximizing firm has supply

$$s(p) = \begin{cases} 0 & p < c_1 \\ [0, \infty) & p = c_1 \\ \infty & p > c_1 \end{cases}$$

Its inverse supply is horizontal, constantly equal to c_1 for all positive levels of output.

6.14. Exercises

6.14.1. Group A

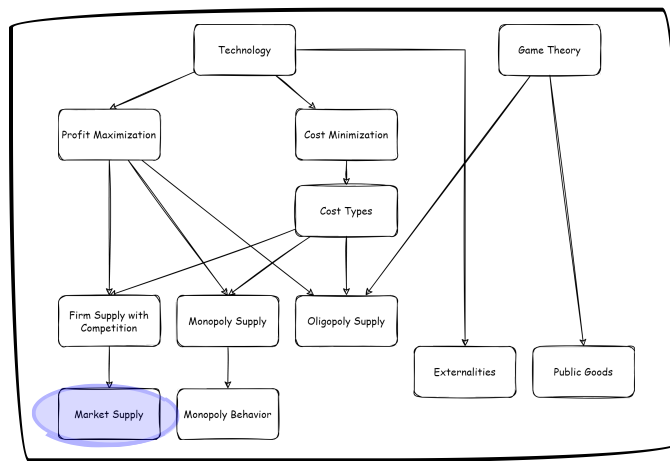
- Consider a price-taking, profit maximizing firm with cost function $c(q) = 1000 + 10q^2$.
 - Calculate the supply function.
 - Find the output level that minimizes the average cost.
 - Calculate the profit function.
 - What is the marginal effect of a price change on profit?
 - What is the minimum price for which the firm makes non negative profit?
- Suppose that a firm has a supply function given by $s(p) = 100 + 4p$. Calculate the inverse supply function.

7. Market Supply

7.1. Context

- Prices and traded quantities in any market are predominantly determined by demand and supply. Our understanding of supply is based on individual firm behavior.
- Nevertheless, most empirical estimations, analyses, and media discussions focus on markets instead of particular firms.
 - How do we consolidate individual firm production decisions into market supply?
 - Why do most popular discussions focus on a market level?
 - Why do we frequently rely on econometrically modeling markets as being competitive?

7.2. Course Structure Overview



7.3. Lecture Structure and Learning Objectives

Structure

- European Common Agriculture Policy (Case Study)
- An Aggregation Exercise
- Basic Concepts
- An Empirical Market Estimation Example
- Current Field Developments

Learning Objectives

- Describe how individual firms' supplies are aggregated market supply.
- Illustrate aggregation with examples.
- Explain why successful policy interventions at a market level are challenging.
- Describe the ideal market structure of perfect competition.
- Provide an empirical example of using market models to estimate price effects on supply.

7.4. European Common Agriculture Policy

- In the late 1990s and early 2000s, the CAP aimed to maintain the status of European agriculture.
- The EU used the minimum intervention price as its main policy instrument.
- The EU's goal was to maintain high demand for agricultural products by guaranteeing suppliers minimum prices for their products.

7.4.1. Impact on the EU Markets

- The CAP reduced production risks.
 - There was less uncertainty about the lower end of price distributions.
- This incentivized suppliers to produce more as they could calculate their production's minimum profit.
- It led to oversupplying certain CAP-supported agricultural products.
- The main goal of the CAP policy was achieved.
- Prices of many agricultural products remained (relatively) high during this period.
- The profitability of the agricultural sector was maintained, and the sector did not shrink.

7.4.2. Butter Mountains

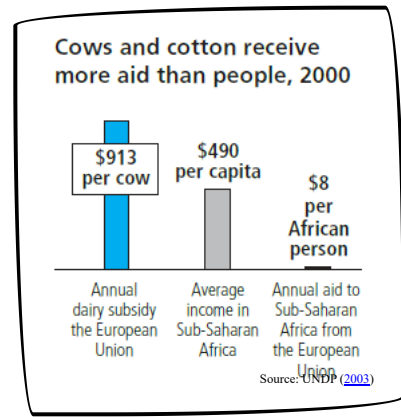
- However, the EU had to purchase and store large quantities of produced agricultural products on many occasions.
- This led to the creation of *butter mountains*, *milk lakes*, and *wine lakes*.
- The alcohol/wine stock peaked at **352** million liters in 2007.



Source: [wikipedia](https://www.wikipedia.org/)

7.4.3. Impact Outside the EU

- How to deal with the stock?
- Some products (e.g., diaries) were exported using subsidies to low-income countries.
- Farmers in developing countries could not keep up with cheap (subsidized) competition from the EU.
- The UNDP's 2005 Human Development Report suggests that the main difficulty in the WTO agriculture negotiations was rich country subsidies.



7.4.4. The Animal Spirits

- Taming the market forces is a very challenging problem.
- On many occasions, policy interventions fail despite the best of intentions.
- Even if a market intervention succeeds in the desired policy direction, there is a serious probability of unexpected side effects.

7.5. A Practical Aggregation Example

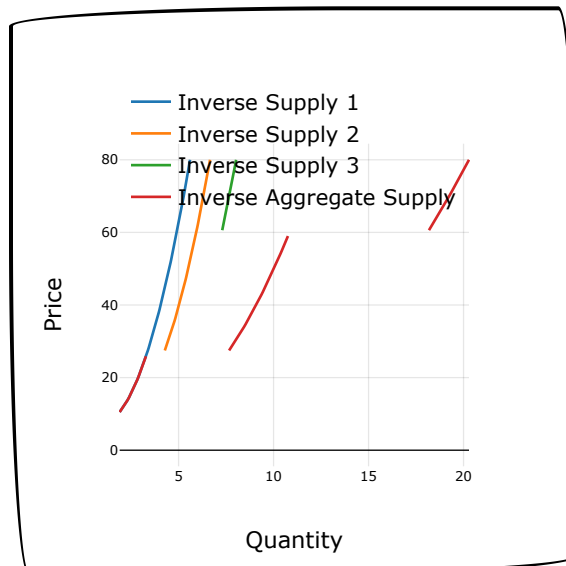
- Suppose **3** firms are producing bicycles according to the supplies in the following table.
- What is the aggregate supply of bicycles in the market?

Price	Firm 1 Supply	Firm 2 Supply	Firm 3 Supply	Market Supply
50	4	7	10	21
100	5	9	20	34
150	6	11	30	47
200	7	13	35	55
250	8	14	40	62
300	9	15	45	69

7.6. Market Supply Aggregation

- For a given price level, the **market supplied quantity** is obtained by aggregating the supplied quantities of all firms in the market.
- The **market supply** function is the sum of the firms' supply functions for each price level.
- The **inverse market supply** function is the inverse of the market supply. Namely, a mapping that gives a market price corresponding to each aggregate supplied quantity.

7.7. A Graphical Aggregation Example



7.8. Market Equilibrium

- We say that a market is in **equilibrium** if buyers and sellers have no incentive to deviate from their choices.
- There is no tendency to change either the price or the bought and sold quantities.

7.9. Competition and Equilibria

A **competitive equilibrium** is an equilibrium where

- there are many consumers willing to buy at the lowest price,
 - there are many producers willing to sell at the highest price, and
 - competition eliminates bargaining power.
- In other words, demand is equal to supply, and the market clears.

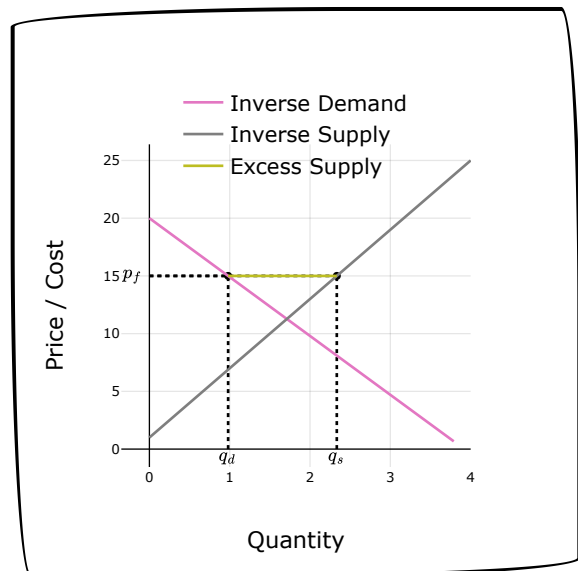
7.10. Non-Equilibrium States

- On some occasions, markets may fail to clear due to either market failures or policy interventions.
- Two potential states that can occur.
 - The price is too high and supply is greater than demand.
 - The price is too low and demand is greater than supply.

7.10.1. Excess Supply

An **excess supply** or **market surplus** is a market state where the supplied quantities exceed the demanded quantities.

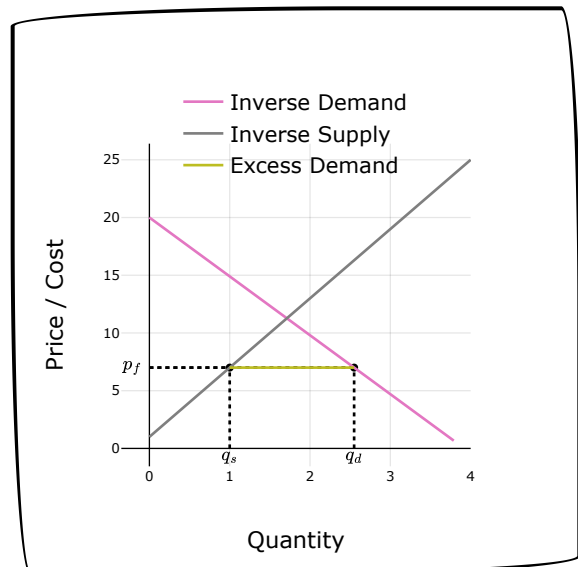
- Unemployment in labor markets.
- Agricultural surpluses (E.g., butter mountains).
- Surpluses due to price controls (price floors) with too high lower thresholds.



7.10.2. Excess Demand

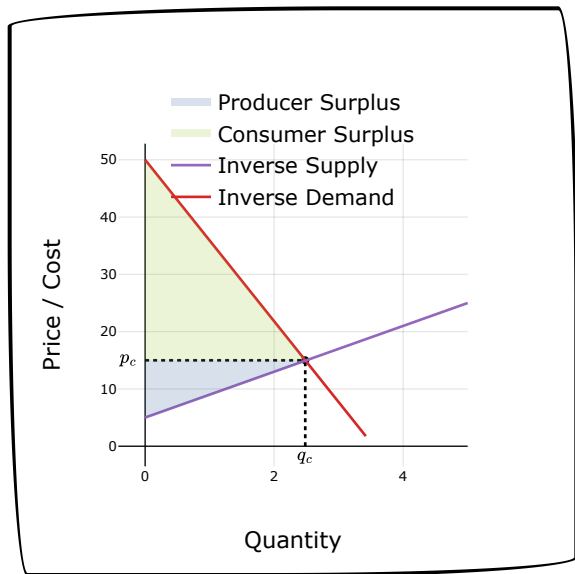
An **excess demand** or **market shortage** is a market state where the demanded quantities exceed the supplied quantities.

- Credit rationing in financial markets.
- The 2021 semiconductor shortage due to the COVID pandemic.
- Shortages due to price controls (price ceilings) with too low upper thresholds.



7.11. The Welfare Properties of Competitive Equilibria

- Suppose inverse demand is given by $p_d(q) = d_0 + d_1 q$ for $d_0 > 0, d_1 < 0$.
- Suppose inverse supply is given by $p_s(q) = s_0 + s_1 q$ for $s_0, s_1 > 0$.
- The market clears, i.e., $p_c = p_d(q_c) = p_s(q_c)$
- How can we calculate the total welfare?



7.12. Market Entry

- A market satisfies the **free entry** condition if no restrictions (legal, technological, etc.) prevent new firms from entering the market.
- A market has **entry barriers** if any form of restriction prevents new firms from operating in the market.

7.13. Profits in the Ideal Case of Free, Instantaneous Entry

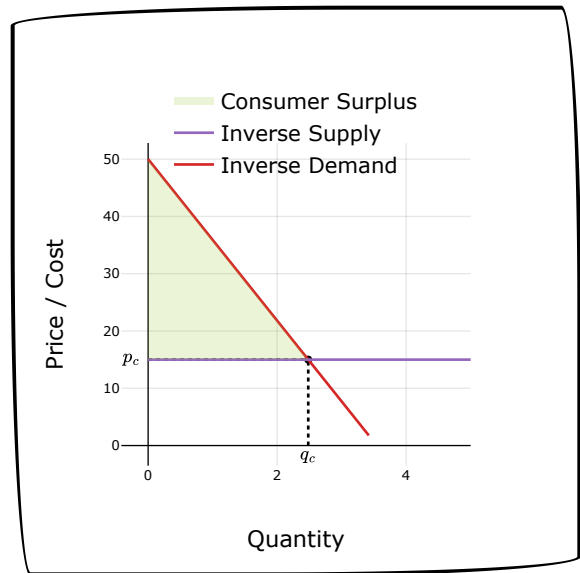
- Suppose that there is a pool of potential firms that can enter a market.
- When does a potential firm have incentives to enter the market?
- Can firms with costlier technologies survive in the market in the long run?
- When does an incumbent firm leave the market?
- An equilibrium in such a market (i.e., a situation for which there is no new entry or exit from the market) can only be achieved if
 - all firms produce at the same marginal cost (i.e., using the same technology) and
 - all firms have zero profits.

7.14. Perfect Competition

- A market is **perfectly competitive** if it has a large number of consumers and firms such that
 - the market does not suffer from any market failure (imperfect information, externalities, etc.),
 - there are no entry barriers in the market,
 - consumers and firms are price takers,
 - consumers try to maximize their utility,
 - firms produce a homogeneous commodity or service,
 - firms try to maximize their profits.

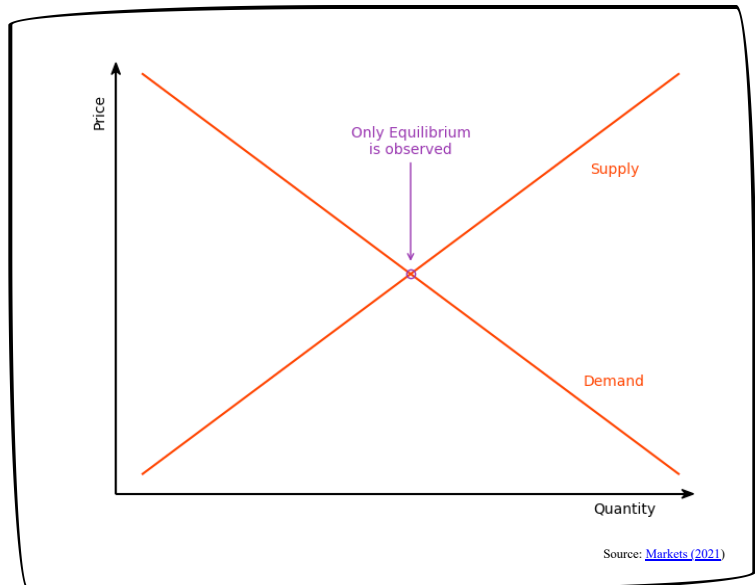
7.15. The Welfare Properties of Perfect Competition

- Suppose inverse demand is given by $p(q) = p_0 + p_1 q$ for $p_0 > 0, p_1 < 0$.
- Inverse supply is flat in perfect competition (why?).
- How can we calculate the total welfare?



7.16. Estimating Market Supply

- Suppose we consider estimating a market in equilibrium.
- In the data,
 - we observe traded quantities and market prices but
 - we do not observe demanded and supplied quantities.



7.16.1. Simultaneity

- One major difficulty in estimating the supply (or demand) equation is simultaneity.
- In market estimations, it boils down to the market clearing condition.

$$\text{Demanded Quantity} = \text{Supplied Quantity} = \text{Traded Quantity}$$

- Prices affect demanded quantities →
- Demanded quantities affect traded quantities →
- Traded quantities affect supplied quantities →
- Supplied quantities affect prices.
- In short, prices are endogenous.

7.16.2. Least Squares

- We know from econometrics that a fundamental requirement for linear regressions is that the independent variables (right-hand side) should be exogenous.
- If we try to estimate

$$\text{Supply}_{it} = \beta_0 + \beta_1 \text{Price}_{it} + \dots + u_{it},$$

we will not obtain the correct coefficient for β_1 because the exogeneity condition is not satisfied.

- This can be a real problem if we want to calculate the elasticity of supply.
- What can we do?

7.16.3. Full Information Maximum Likelihood

- We should take into account the complete market system.
- One way to do it is via a methodology called full information maximum likelihood.
- Without many details, the process
 1. models the market as a system of simultaneous equations, and
 2. estimates the demand and supply equations together.
- Luckily there is statistical software that automates this process.

7.16.4. Market Estimation Using R

- Install and load the R package markets (<https://cran.r-project.org/package=markets>).

```
install.packages('markets')
library(markets)
```

- Read the description of the `houses` dataset (<https://markets.pikappa.eu/reference/houses.html>).
- Estimate and summarize the supply equation using a linear regression.

```
ls <- lm(HS ~ RM + TREND + W + L1RM + MA6DSF + MA3DHF + MONTH, fair_houses())
summary(ls)
```

- Estimate and summarize the market system in equilibrium.

```
eq <- equilibrium_model(
  HS | RM | ID | TREND ~
  RM + TREND + W + CSHS + L1RM + L2RM + MONTH |
  RM + TREND + W + L1RM + MA6DSF + MA3DHF + MONTH,
  fair_houses(), correlated_shocks = FALSE)
summary(eq)
```

- Compare the two supply price coefficients. Are both of them statistically significant?

7.16.5. Market Visualization Using R

- Plot the data points, the average demand, and supply equations on the quantity-price plane from the estimates of the equilibrium model.

```
plot(eq)
```

- Add the regression line to the plot.

```
abline(ls)
```

- For which estimate is supply more elastic?

7.17. Current Field Developments

- Methods for market estimation is an ongoing research topic.
- There are many methods focusing on estimating different market aspects and types, such as
 - complementary systems of demands ([Deaton and Muellbauer 1980](#); [MicEconAids 2017](#)),
 - differentiated product markets ([Berry, Levinsohn, and Pakes 1995](#); [BLPestimator 2019](#)),
 - shortages and surpluses ([Maddala 1986](#); [Markets 2021](#)).
- Structural macroeconomic models frequently entail aggregate demand and supply for the national and, sometimes, international economies.

7.18. Concise Summary

- Market supply consolidates the supplies of all firms in a market.
- Policy-makers often target market supply.
- Controlling market forces can be challenging and have many unexpected side effects.
- Many estimation methods are available for estimating demand and supply.
- Competitive equilibria are Pareto efficient.
- Perfect competition, although unrealistic, offers us a benchmark of how markets could ideally operate.

7.19. Further Reading

- Varian ([2010, secs. 24.1-4](#))
- CORE Team ([2017, secs. 8.2, 8.4, 8.5, 8.8](#))

7.20. Mathematical Details

Suppose there are n firms in the market with supply functions $s_1, \dots, s_n$. The market supply is given by

$$s(p) = \sum_{i=1}^n s_i(p).$$

7.20.1. The Rate of Change of Aggregate Supply with Respect to Price

The rate of change of aggregate supply is the sum of the rates of change of the individual supplies of each firm in the market. This is a direct consequence of the linearity of differentiation.

$$s'(p) = \sum_{i=1}^n s'_i(p) > s'_j(p) \quad (\forall j).$$

7.20.2. Elasticity of Aggregate Supply

The elasticity of aggregate supply is a weighted average of the elasticities of the individual firms.

$$\begin{aligned} e_s(p) &= s_p(p) \frac{p}{s(p)} \\ &= \left(\sum_{i=1}^n s'_i(p) \right) \frac{p}{s(p)} \\ &= \sum_{i=1}^n s'_i(p) \frac{p}{s_i(p)} \frac{s_i(p)}{s(p)} \\ &= \sum_{i=1}^n e_{s_i}(p) \frac{s_i(p)}{s(p)} \end{aligned}$$

The weights are given by the market shares $\frac{s_i(p)}{s(p)}$.

7.21. Exercises

7.21.1. Group A

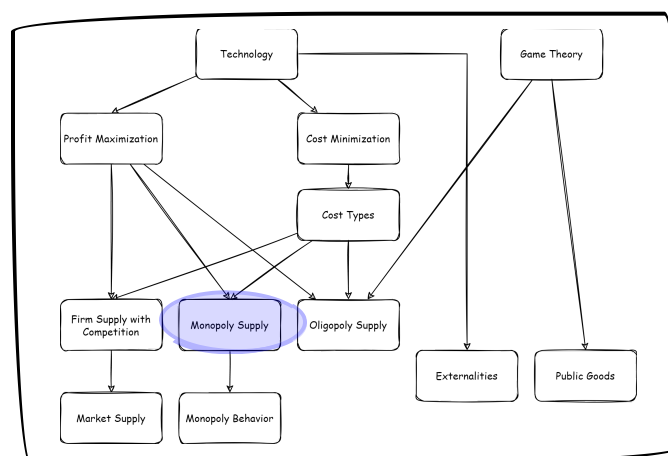
- Suppose that a market consists of two firms, one with supply $s_1(p) = -10 + p$ and one with $s_2(p) = -15 + p$. Calculate the market supply function. At which prices does the market supply function have kinks?
- Consider a market with multiple price-taking firms, all of which have cost functions $c(q) = q^2 + 1$ for $q \geq 0$. Suppose that entry is free and instantaneous in the market, the market demand function is $d(p) = 52 - p$, and the market perpetually clears.
 - Calculate the supply and profit functions of an individual firm.
 - Assume that n is a natural number (positive integer). What is the market supply function if there are n identical firms in the market?
 - What does market-clearing imply for the relationship between the number of firms and the market price?
 - Use the last condition to write the individual firm's profit as a function of the number of firms in the market.
 - Find the equilibrium number of firms.
 - What will be the equilibrium supplied quantity in the market?
 - Suppose that demand shifts to $d(p) = 52.5 - p$. Calculate the equilibrium number of firms and the equilibrium supplied quantity. Are firms' profits zero in this case?

8. Monopoly

8.1. Context

- The goal in the popular economic-themed board game "Monopoly" is to monopolize a fictional property market by driving other players to bankruptcy.
- In real markets, however, monopolies tend to have unpleasant connotations. In fact, the competition policies in both the US and EU aim (among others) to prevent the growth and abuse of monopoly power.
 - Why do regulators try to reduce monopoly power?
 - Are monopolies bad in terms of economic efficiency?
 - How do monopolies choose prices for their products?

8.2. Course Structure Overview



8.3. Lecture Structure and Learning Objectives

Structure

- The Baby Bells (Case Study)
- Basic Concepts
- An Example
- Welfare Analysis of Monopolies
- Current Field Developments

Learning Objectives

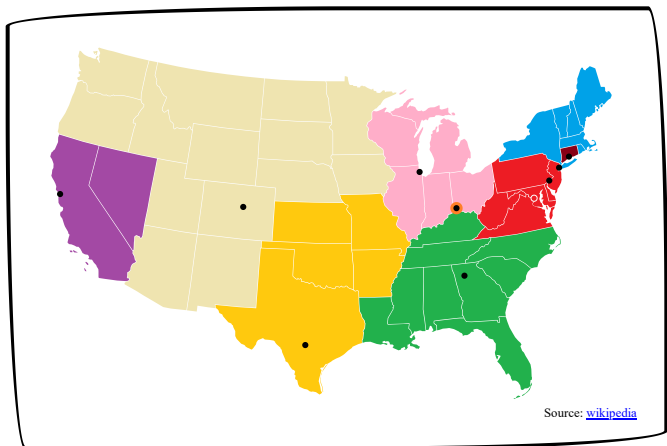
- Describe the monopoly market structure.
- Illustrate how monopolies make decisions to maximize their profits.
- Compare and contrast the welfare of competition and monopoly market structures.
- Explain the economic inefficiencies introduced by profit maximizing monopolies.
- Describe the conditions under which monopolies may arise.

8.4. The Baby Bells

- In 1974, the US Department of Justice filed an antitrust lawsuit against AT&T.
- AT&T was the sole provider of telephone services in the US.
- Its subsidiary, Western Electric, produced most telephonic equipment.
- Overall, AT&T had almost wholly controlled the communication technology sector in the US.

8.4.1. The AT&T Breakup

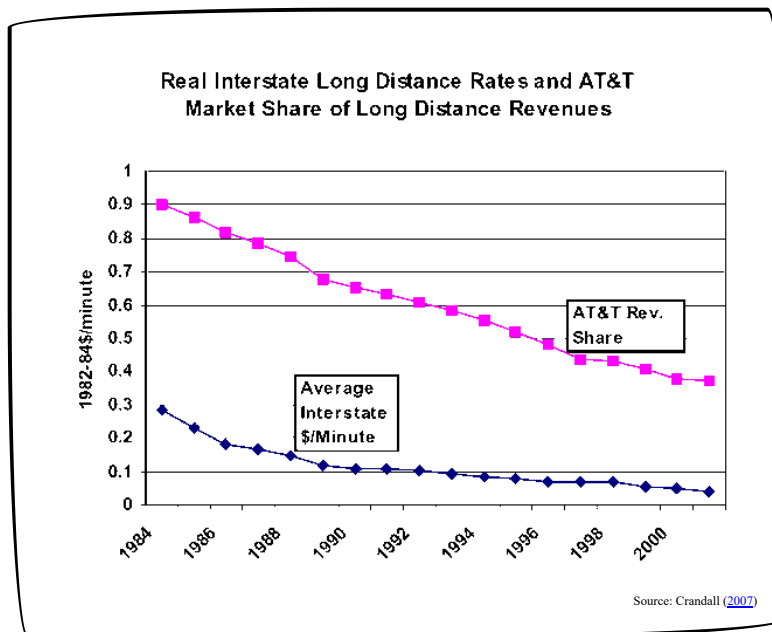
- In 1982, a settlement between AT&T and the US Department of Justice was reached.
- The main provision in the settlement was the breakup of AT&T into seven independent companies.
- Two additional partial subsidiaries of AT&T were acquired by other companies after the breakup.
- These regional companies became known as Baby Bells.



8.4.2. AT&T Market Power

- The US vs. AT&T case (1982) is perceived as one of the most successful US antitrust cases.
- The US Department of Justice argued that AT&T's market power led to higher prices and fewer services.
- The AT&T monopoly accounted for **80 – 85%** of telephone lines in 1982.
- After the breakup, prices fell, and more services were provided.

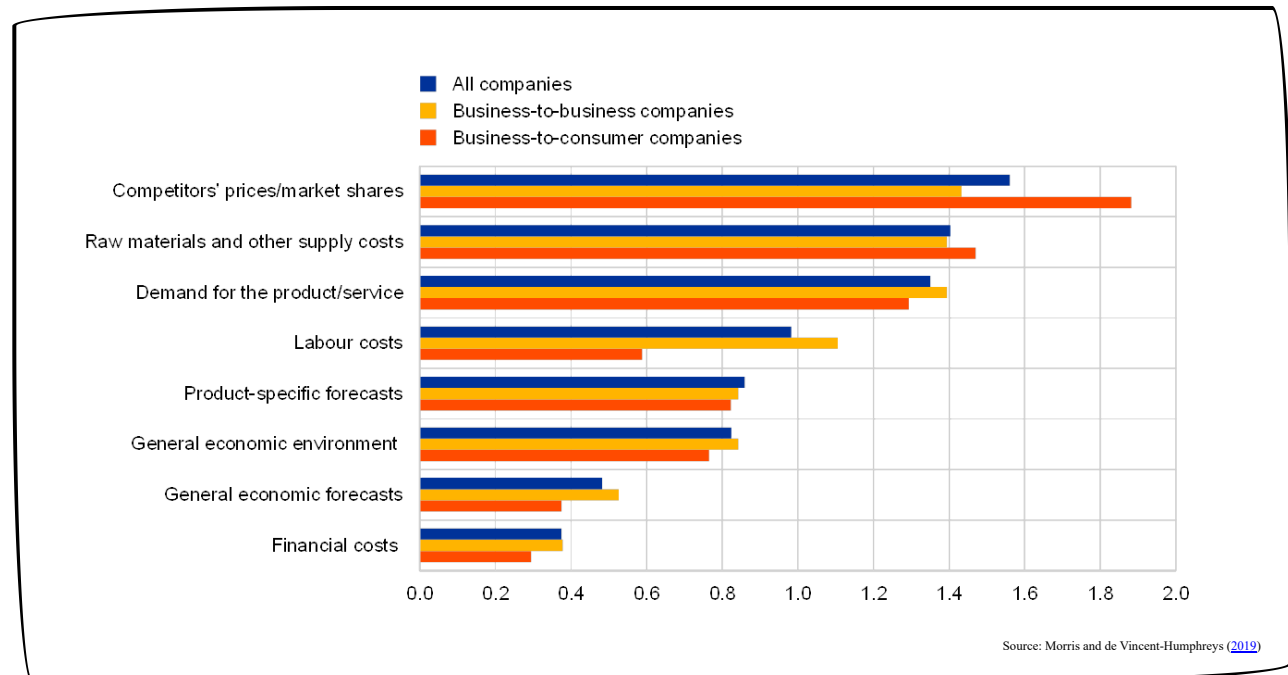
8.4.3. Prices After the AT&T Breakup



8.4.4. AT&T Price Setting

- The AT&T case does not seem to fit well the price taking behavior of firms in perfect competition.
- Was the influence of prices a special AT&T characteristic?
- What do price setting firms consider when they choose their prices?

8.4.5. What Firms Consider when Setting Prices?



8.5. Monopoly

- Perfect competition requires that a large number (formally an infinite number) of firms (sellers) exist in the market.
- What about the other extreme case of a single firm in a market?
- A market structure with exclusive possession of supply by a single seller is called a **monopoly**. The single firm (or seller) in a market is called a **monopolist**.
- Market demand and firm demand are identical in monopolistic markets.

8.6. Price Setting

- Is price taking behavior an appropriate assumption for monopolies?
- The justification for price taking is based on competition.
- Attempts to change prices do not work because other firms do not follow them.

- However, this is not a valid argument in a single seller market.
- A firm (or a consumer) is a **price setter** if it can influence the market price of the products it produces. Price setters consider market prices as (at least partially) endogenous.

8.7. An Example

- Suppose that the monopolist's cost function is

$$c(q) = 2q^2 + 2.$$

- Let the inverse market demand be

$$p(q) = 70 - 3q$$

- The monopolist wants to maximize its profits

$$\max_q \{ (70 - 3q)q - (2q^2 + 2) \}.$$

8.7.1. The Profit Maximization Condition

- The monopolist wants to produce a quantity level for which its marginal cost equals its marginal revenue.
- If marginal cost is greater than marginal revenue, then the monopolist can increase its profit by reducing production.
- If marginal revenue is greater than marginal cost, then the monopolist can increase its profit by increasing production.
- Therefore, at the maximum, we should have

$$70 - 6q = 4q$$

8.7.2. Monopoly Vs. Competition

Market Structure	Quantity	Price
Monopoly	$q_m = 7$	$p_m = 70 - 21 = 49$
Competition	$q_c = 10$	$p_c = 40$

- The monopolist produces less than the competition quantity.
- The monopolistic price is greater than the competition price.

8.7.3. Monopolistic Profit

- The monopolist makes profit

$$\begin{aligned} \pi_m &= p_m q_m - c(q_m) \\ &= 343 - (98 + 2) \\ &= 243. \end{aligned}$$

- In the competitive equilibrium, profit is

$$\begin{aligned} \pi_c &= p_c q_c - c(q_c) \\ &= 400 - (200 + 2) \\ &= 198. \end{aligned}$$

8.8. The Welfare Effects of Monopolies

- We have shown that prices and quantities are different in competition and monopoly.
- How can we measure which situation is better?
- Which situation is in favor of the firm?
- Which situation is in favor of the consumers?
- Which situation is overall better?

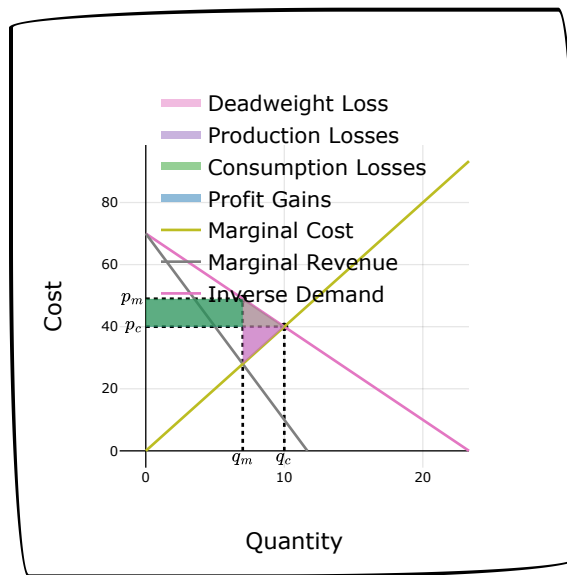
8.8.1. Total Welfare

- The **consumers' surplus** is the consumer benefit from buying a commodity or service at a price lower than her reservation value.
- The **producers' surplus** is the producer's benefit stemming from selling a commodity or service at a price greater than her reservation value.
- The **total welfare** (or economic surplus) is the sum of consumers' and producers' surplus.

8.8.2. Deadweight Loss

- We can compare the total welfare of the two market structures to examine which one is economically more efficient.
- The **deadweight loss** (or excess burden) is a measure of lost economic efficiency compared to when the socially optimal quantity of a commodity or a service is produced.
- The **deadweight loss of a monopoly** is the difference between the competition and the monopoly's total welfare.

8.9. A Deadweight Loss Example

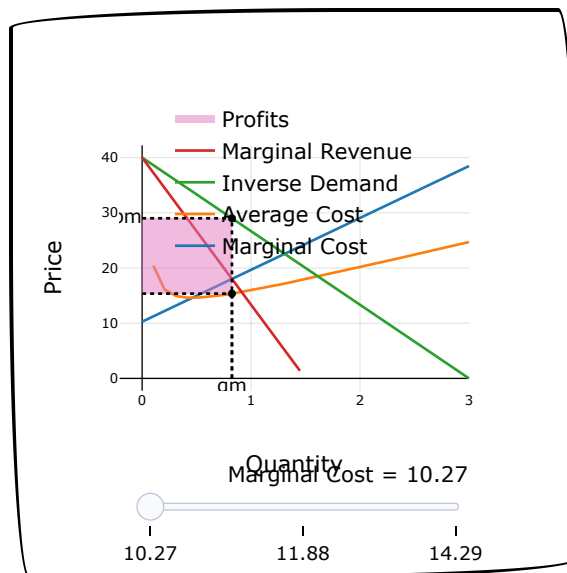


8.10. Markup Pricing

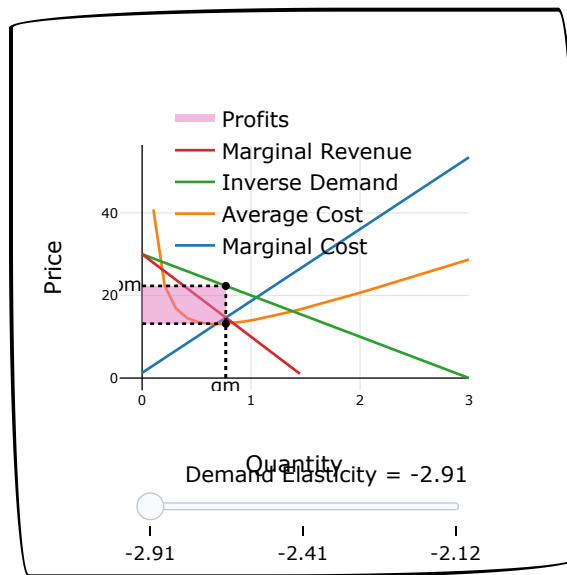
- The monopolist sets the price of the commodity it produces using a markup.
- The monopolist pricing rule can be calculated as

$$p_m = \frac{\text{Marginal Cost}}{\frac{1}{\text{Demand Elasticity}} + 1}$$

8.10.1. How do Marginal Cost Changes Affect Profit?



8.10.2. How do Elasticity Changes Affect Profit?

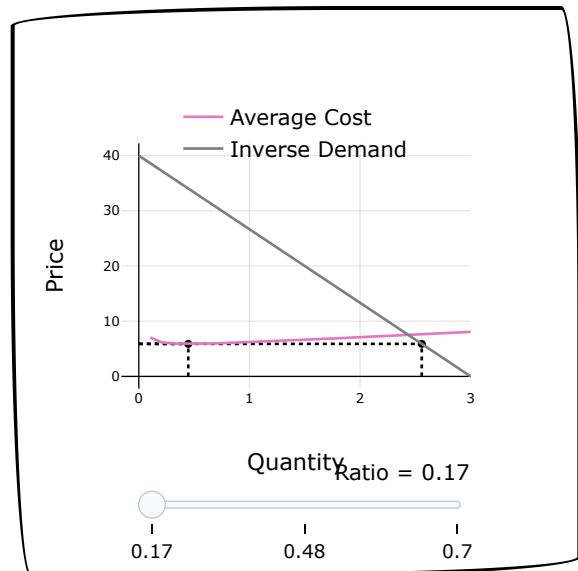


8.11. Natural Monopolies

- A **natural monopoly** is a type of monopoly that manifests because of high start-up costs, technological barriers, or increasing returns of scale.
- Examples of entry barriers that can lead to natural monopolies are:
 - Infrastructure costs such as those of the electricity or water supply networks.
 - High-end technologies such as those needed in the production of semiconductor chips.
 - Exclusive access to a needed production factor.

8.12. When are Monopolies Economically More Likely to Appear?

- The answer depends on both market demand and production cost.
- If demanded quantities are much larger than the minimum average production cost, then there is room for many firms in the market, and a monopoly structure is less likely to appear.
- This can be formalized by looking at the minimum average production cost relative to demand.



8.12.1. Minimum Efficient Scale

- The **minimum efficient scale** is the lowest production scale where a firm minimizes its average cost.

8.13. Current Field Developments

- Antitrust laws and competition policies are well integrated into the US and EU economic systems.
- For instance, the EU has investigated Google on a variety of occasions for breaches of the EU competition laws since 2010.
- Extensions of the monopoly model that we have seen are actively used in economic research.
- Dynamic extensions of the monopoly model are used to study the impact of firm size on R&D.

8.14. Concise Summary

- The monopoly represents the opposite market structure of perfect competition in terms of the number of firms in the market.
- Instead of having many small, price taking firms, a monopoly consists of a single, large, price setting firm.
- Monopolies introduce economic inefficiencies due to their market power.
- They result in deadweight losses for the economy as a whole.
- Entry barriers can foster the creation of (natural) monopolies.

8.15. Further Reading

- Varian (2010, chap. 25)
- CORE Team (2017, secs. 7.5-8)
- Morris and de Vincent-Humphreys (2019)
- Crandall (2007)

8.16. Mathematical Details

Another extreme market structure case is to examine markets that consist of exactly one firm. The single firm is called a monopolist. Price taking behavior is less of a realistic assumption one can make for monopolistic markets. Market demand and firm demand are identical in monopolistic markets.

8.16.1. A Profit Maximizing Monopolist

Using the cost function, the profit maximization problem of a monopolist is given by

$$\max_p \{pd(p) - c(d(p))\}.$$

The cost function incorporates the technological constraints faced by the firm. The profit can be decomposed into two parts, namely

- revenue $pd(p)$ and
- cost $c(d(p))$.

The difference with the competition case stems from the dependence of prices on the supplied quantity via the demand function $d(p)$.

As long as demand is a one-to-one function (i.e., there is a single price for each demanded quantity and vice versa), maximizing over quantities or prices does not affect the optimization outcome. This condition will be satisfied in most healthy markets. The problem is then written using quantities as an optimization control as

$$\max_q \{p(q)q - c(q)\}.$$

For non-boundary solutions, the marginal revenue and marginal cost are equalized at the maximum (why?). With price as the control, this means

$$d(p) + pd'(p) = c'(d'(p))d'(p).$$

With quantity as the control, the variational condition becomes

$$p'_d(q)q + p_d(q) = c'(q),$$

where p_d is the inverse demand. The variational condition is necessary. It gives a maximum if

$$p''_d(q)q + 2p'_d(q) - c''(q) < 0.$$

1. Markup Pricing

The optimal price is set by

$$p = \frac{c'(d(p))}{\frac{1}{e_d(p)} + 1}.$$

2. Market Power

The market power of the monopolist depends on the elasticity of demand. From the variational condition with price as the control, we get

$$\begin{aligned} c'(d(p)) &= \frac{d(p)}{d'(p)} + p \\ &= p \frac{d(p)}{p} \frac{1}{d'(p)} + p \\ &= p \left(\frac{1}{e_d(p)} + 1 \right) \end{aligned}$$

The greater is e_d (i.e., the smaller is the absolute value of the elasticity), demand becomes more inelastic, and the monopolist's profit increases.

3. Welfare Analysis

Market power usually introduces inefficiencies in the market structure. The monopolist produces less than the competitive quantity and sells at a price higher than the competitive price. How can we measure the economic loss? Let p_m and p_c correspondingly denote the monopoly and competition prices.

The total welfare in the competition case can be calculated by

$$\begin{aligned} W(q_c) &= \Pi_c(q_c) + \Pi_s(q_c) \\ &= \int_0^{q_c} (p(q) - c'(q)) dq \\ &= \int_0^{q_c} p(q) dq - \mu(q_c). \end{aligned}$$

Similarly, the total welfare in the monopoly case is given by

$$W(q_m) = \int_0^{q_m} p(q) dq - \mu(q_m).$$

The difference is called deadweight loss

$$\begin{aligned} D(q_c, q_m) &= W(q_c) - W(q_m) \\ &= \int_{q_m}^{q_c} p(q) dq - [\mu(q_c) - \mu(q_m)]. \end{aligned}$$

8.16.2. Minimum Efficient Scale

The minimum efficient scale is the level of output that minimizes average cost, namely

$$\text{MES} = \arg \min_q \bar{c}(q).$$

The greater is the ratio of the minimum efficient scale to the demanded quantity corresponding to the price being equal to the minimum average cost, i.e.,

$$\frac{\text{MES}}{d(\bar{c}(\text{MES}))},$$

the more possible become monopolistic structures.

8.16.3. An Affine Demand and Cost Example

For this example let the cost function be given by

$$c(q) = c_1 q + c_0 \quad (c_1, c_0 > 0).$$

Let also the inverse demand function be

$$p(q) = p_0 + p_1 q \quad (p_0 > 0, p_1 < 0).$$

1. Monopoly and competition quantities

For the monopoly, some simple calculations give

$$q_m = \frac{c_1 - p_0}{2p_1}.$$

For the competition case, we have

$$q_c = \frac{c_1 - p_0}{p_1}.$$

Solutions are valid for $p_0 > c_1$ because otherwise, the quantities become negative.

2. How do marginal cost changes affect profit?

The monopolist's profit is

$$\begin{aligned} \pi_m &= p(q_m)q_m - c(q_m) \\ &= -\frac{(c_1 - p_0)^2}{4p_1} - c_0. \end{aligned}$$

The profit decreases as marginal cost (here c_1) increases

$$\frac{d\pi_m}{dc_1} = -\frac{c_1 - p_0}{2p_1} < 0.$$

3. How do elasticity changes affect profit?

The elasticity of demand at the monopolistic quantity is

$$e_d = \frac{1}{p_1} \frac{p_1 q_m + p_0}{q_m} = \frac{c_1 + p_0}{c_1 - p_0}.$$

Keeping the marginal cost constant, elasticity changes with p_0 . That is

$$\frac{\partial e_d}{\partial p_0} = \frac{2c_1}{(c_1 - p_0)^2} > 0.$$

Therefore, profit increases as the elasticity e_d increases ($|e_d|$ decreases and demand becomes more inelastic), i.e.,

$$\begin{aligned} \frac{d\pi_m}{de_d} &= \frac{d\pi_m}{dp_0} \frac{\partial p_0}{\partial e_d} \\ &= \frac{c_1 - p_0}{2p_1} \frac{(c_1 - p_0)^2}{2c_1} > 0. \end{aligned}$$

4. Deadweight loss

The deadweight loss is

$$\begin{aligned}
D(q_c, q_m) &= \int_{q_m}^{q_c} [p(q) - c'(q)] dq \\
&= \int_{q_m}^{q_c} [p_0 + p_1 q - c_1] dq \\
&= \left[(p_0 - c_1)q + \frac{p_1}{2} q^2 \right]_{q_m}^{q_c} \\
&= -\frac{(c_1 - p_0)^2}{8p_1}.
\end{aligned}$$

8.16.4. An Affine Demand, Cubic Cost Example

The calculations of this example are used for producing the figures of sections [8.10.1](#) and [8.10.2](#). Let the cost function be given by

$$c(q) = q^3 - 2q_2 q^2 + (q_2^2 + q_1)q + q_0 \quad (q_2, q_1, q_0 > 0).$$

Let also the inverse demand function be

$$p(q) = p_1 q + p_0 \quad (p_0 > 0, p_1 < 0).$$

1. Monopoly and competition quantities

For the monopolistic market structure, some tedious calculations give

$$\begin{aligned}
p'(q)q + p(q) &= c'(q) \implies \\
q_m &= \frac{1}{3} \left(2q_2 + p_1 + \sqrt{q_2^2 - 3q_1 + p_1^2 + 4q_2 p_1 + 3p_0} \right).
\end{aligned}$$

For the competition case, we have

$$\begin{aligned}
p(q) &= c'(q) \implies \\
q_c &= \frac{1}{6} \left(4q_2 + p_1 + \sqrt{4q_2^2 - 12q_1 + p_1^2 + 8q_2 p_1 + 12p_0} \right).
\end{aligned}$$

8.17. Exercises

8.17.1. Group A

1. Consider a monopolist with cost function $c(q) = 2q$. Suppose that market demand is given by $d(p) = 100 - 2p$. Calculate the monopolist's price, quantity, and profit. In addition, calculate the deadweight loss.
2. Consider a monopolist with cost function $c(q) = q^2$. Suppose that market demand is given by $d(p) = 100/p$. Calculate the quantity that the monopolist would like to produce.
3. Consider a monopolist operating in a market where the commodity's demand depends on a qualitative attribute of the product besides its price. Specifically, demand is given by $d(s, p) = 3\sqrt{s}/p^3$, where s is the qualitative attribute of the market's commodity. The cost function of the monopolist also directly depends on the qualitative attribute, i.e., $c(s, q) = s/2 + 2q$. The monopolist's goal is to maximize profit by simultaneously choosing both the qualitative attribute level and the commodity's price. Calculate the monopolist price, the optimal level of the qualitative attribute, the output level, and the profit.
4. Can the monopolist select a price level for which demand is inelastic (i.e., $0 > e_d > -1$)?
5. Suppose that the elasticity of demand $e_d < -1$ and the marginal production cost $c_1 > 0$ are constant. Calculate the marginal effect on prices of an improvement in the production technology reducing the marginal cost in a monopolistic market.

8.17.2. Group B

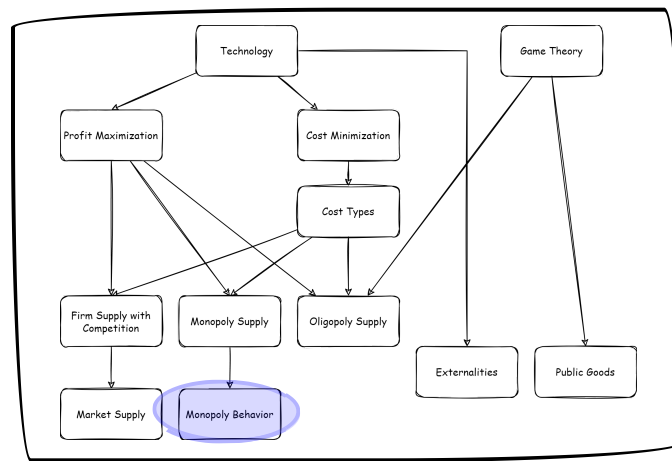
1. Consider a monopolist with cost function $c(q) = c_1 q$. Suppose that market demand is given by $d(p) = a/p^b$ for $a > 0$ and $b \geq 1$. Calculate the monopolist's price, quantity, and profit. In addition, calculate the deadweight loss for $c_1 = 2$, $a = 16$, and $b = 2$.
2. Show that if demand is affine and the marginal cost is constant, the rate of change of prices with respect to marginal cost in a monopolistic market is $1/2$.

9. Monopoly Behavior

9.1. Context

- Many market structures examined in economics assume that the market has a single price. Both perfect competition and monopoly, which we have previously studied, are single price market structures.
- However, in practice prices for similar, and in some cases even identical products, can be different from seller to seller.
 - Why do we observe price dispersion in real markets?
 - How is this related to the firms' choices?
 - How does this affect the total market welfare?

9.2. Course Structure Overview



9.3. Lecture Structure and Learning Objectives

Structure

- Marseille and Ancona Fish Market (Case Study)
- Basic Concepts
- An Example of Group Pricing
- Welfare Analysis of Price Discrimination
- Current Field Developments

Learning Objectives

- Describe the pricing strategies available to the monopolist.
- Explain the difference between price discrimination and price dispersion.
- Illustrate price discrimination by a group pricing example.
- Analyze the welfare effects of price discrimination.
- Describe product differentiation and monopolistic competition.

9.4. Marseille and Ancona Fish Markets

- Price differences might arise due to various reasons.
- A potential reason is price discrimination. But...
- Price dispersion is not necessarily due to price discrimination.

9.4.1. Market Structure Matters

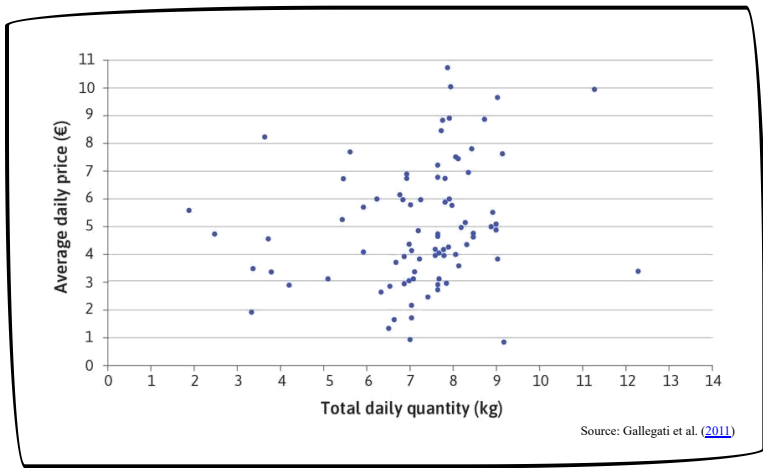
- Marseille:
 - Sellers have stands.
 - Buyers approach the stands.
 - Prices are pairwise agreed.
- Ancona:
 - Fish crates go down a conveyor belt.
 - A screen displays the initial price. An auctioneer decides the initial price.
 - The price declines incrementally until a buyer pushes a button to buy.
- Gallegati et al. (2011) studied prices in these two markets.

9.4.2. Price Discrimination in Marseille

- Individual vendors vary prices of the same kind of fish to different buyers by up to **30%**.
- Some buyers are loyal to certain sellers, while others circulate.
- More loyal customers pay slightly higher prices than their less loyal counterparts.
- Sellers prioritize the demand of more loyal customers.
- This market structure leads to higher profits for sellers and higher payoffs (counting demand satisfaction) for **90%** of loyal customers.

9.4.3. Price Dispersion in Ancona

- Individual buyers in the market have very atypical price-quantity relationships.
- Even without face-to-face interaction, buyer loyalty is present.
- Many loyal customers pay lower prices than their less loyal counterparts.



9.5. Market Power

- **Market power** refers to the ability of a firm to influence the price at which it sells the product or service it offers.
- Monopolies tend to have market power, which they can use to increase their profits.

9.6. Reservation Values

- A **reservation value** is the least favorable price point at which an economic agent is willing to engage in trade.
- Examples:
 - A **buyer's reservation value** is the maximum price she is willing to pay in exchange for a commodity or service.
 - A **seller's reservation value** is the minimum price she is willing to sell a commodity or service.

9.7. Price Discrimination

- Conventional Definition: Price discrimination is present when a producer sells the same commodity at different prices to different consumers.
- But this could be due to differences in shipping costs etc.
- Preferred Definition by Stigler (1987): **Price discrimination** occurs when a producer sells two or more similar goods at prices that have different markups.
- Example: A book that sells in hardcover for **15 €** and paperback for **5 €**.

9.7.1. Types of Price Discrimination

- The classic typology from Pigou (1920), distinguishes three price discrimination classes.
- Their names are not very helpful: first, second, and third price discrimination degree.
- We follow the naming convention of Belleflamme and Peitz (2010).
 - Group Pricing (aka third-degree price discrimination)
 - Personalized pricing (aka first-degree or perfect price discrimination)
 - Menu Pricing (aka second-degree price discrimination, or nonlinear pricing)

9.7.2. Group Pricing

- **Group pricing** occurs when a seller offers different markups to different groups of consumers.
- This is perhaps the most common form of price discrimination.
- Example: Student discounts.

9.7.3. Menu Pricing

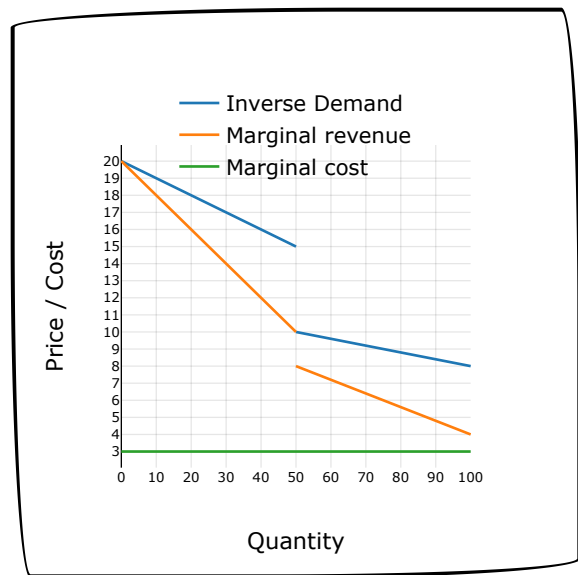
- **Menu pricing** occurs when a seller offers different markups depending on the characteristics of the enclosing package bought, but not across consumers.
- For instance, packages (or menus) may differ in the number of units of the good they contain.
- Quantity discounts or premia are the most prominent examples, but there are also other cases...
- Some airlines add surcharges for one-way tickets.

9.7.4. Personalized Pricing

- **Personalized pricing** occurs when a seller sets the price of each sold unit equal to the maximum amount that the consumer who is buying it is willing to pay for that unit.
- The reservation value of each purchased unit must be known to the seller.
- In reality, impractical due to the enormous amount of required information.
- It can be thought of as an extreme case of group pricing.

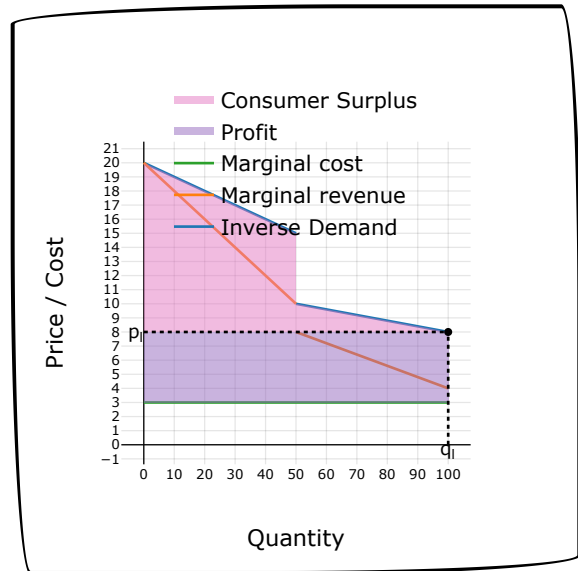
9.8. A Group Pricing Example

- Under uniform pricing, the monopolist can charge a single price.
- Under price discrimination, the monopolist can charge two different prices.
- Let demand be (piecewise) affine and cost be linear (no fixed cost).



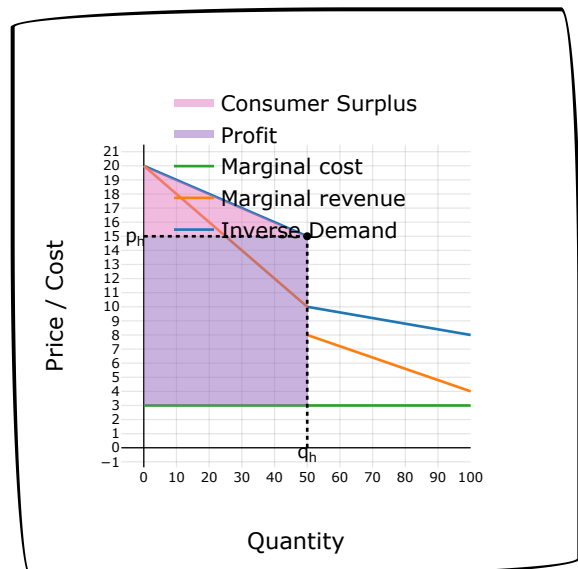
9.8.1. Low Uniform Pricing

- At $p_l = 8$, all customers are buying
- $q_l = 100$
- $\pi_l = q_l (p_l - c) = 500$
- $W_l^c = \frac{50 \times 19}{2} + \frac{50 \times 2}{2} = 525$
- $W_l = \pi_l + W_l^c = 1025$



9.8.2. High Uniform Pricing

- At $p_h = 15$, only high spenders are buying
- $q_h = 50$
- $\pi_h = q_h (p_h - c) = 600$
- $W_h^c = \frac{50 \times 6}{2} = 125$
- $W_h = \pi_h + W_h^c = 725$

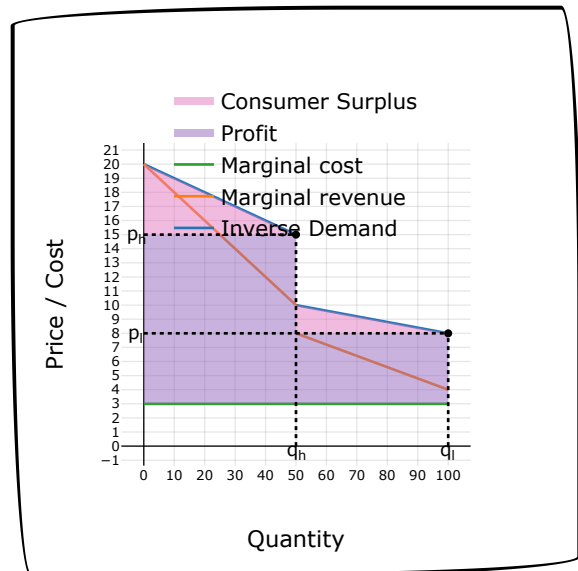


9.8.3. Price Discrimination

- The firm charges $p_{g,h} = 15$ to high spenders and $p_{g,l} = 8$ to low spenders
- $q_{g,h} = 50$ and $q_{g,l} = 50$
- Profit

$$\begin{aligned}\pi_g &= q_{g,h}(p_{g,h} - c) + q_{g,l}(p_{g,l} - c) \\ &= 600 + 250 = 850\end{aligned}$$

- $W_g^c = \frac{50 \times 5}{2} + \frac{50 \times 2}{2} = 175$
- $W_g = \pi_g + W_g^c = 1025$



9.9. Product Differentiation

- Another potential strategy for a firm to gain market power is to attempt to make its product distinct.
- **Product differentiation** refers to firm strategies aiming to distinguish its product from similar products produced by competitors. Product differentiation can be present either due to branding or product quality differences.
- Consumers who perceive a product as being different from competitive products are less likely to substitute it with alternatives.
- Essentially, this makes demand more inelastic, which increases the firms' market power.

9.10. Monopolistic Competition

- **Monopolistic competition** is a market structure with multiple competing firms selling products that are differentiated from one another.
 - The market does not suffer from any other market failure (imperfect information, externalities, etc.).
 - There are no entry barriers in the market.
 - Consumers are price takers, but firms are setting prices.
 - Consumers try to maximize their utility.
 - The differentiated products are not perfect substitutes.
 - Firms try to maximize their profits.

9.11. Current Field Developments

- Firm strategies like product differentiation and price discrimination are actively examined in specific market contexts by economists (e.g., [Gallegati et al. 2011](#)).
- Policy makers are also interested in market models involving such strategies because competition regulation prevents some forms of price discrimination.
- Personalized pricing becomes even more relevant in digital economies where data accumulation about consumers is feasible.
- The recent EU privacy Law affects the availability of pricing strategies of firms operating on the web ([Borgesius and Poort 2017](#)).
- Dynamic extensions of market models indicate that firms can also profit by inter-temporal price discrimination strategies.

9.12. Concise Summary

- Price dispersion is not necessarily price discrimination.
- Firms can employ price discrimination strategies to increase their profits.
- This can increase the total welfare of the market.
- The consumers in the market, however, do not necessarily do better.
- Firms can use product differentiation strategies to increase their market power.
- Monopolistic competition is a market structure that borrows elements from both perfect competition and monopoly.

9.13. Further Reading

- Varian ([2010, chap. 26](#))
- CORE Team ([2017, sec. 11.2](#))
- Gallegati et al. ([2011](#))
- Borgesius and Poort ([2017](#))
- Belleflamme and Peitz ([2010, chaps. 8, 9](#))

9.14. Mathematical Details

9.14.1. Personalized Pricing

Perfect price discrimination can increase the total welfare in cases of monopolies. Not everyone is doing better, though. The monopoly extracts the whole welfare and consumers nothing.

1. Demand and cost

Suppose that demand is given by

$$d(p) = \begin{cases} -\frac{p_0}{p_1} + \frac{1}{p_1}p, & 0 \leq p < p_0, \\ 0, & \neq \end{cases}$$

where $p_0 > 0, p_1 < 0$. Let the cost function be

$$c(q) = c_1 q,$$

with $c_1 > 0$.

2. Uniform Monopoly Pricing

Under uniform pricing the monopolist's objective is

$$\max_p \{pd(p) - c(d(p))\} = \max_p \{d(p)(p - c_1)\}.$$

We obtain the first order condition

$$p_0 - 2p_m + c_1 = 0,$$

which gives the optimal price

$$p_m = \frac{p_0 + c_1}{2} \Rightarrow q_m = -\frac{p_0 - c_1}{2p_1}$$

We can calculate the welfare in this case by adding the consumer's and producer's surpluses. The producer's surplus is equal to profit because the cost function does not have a fixed cost component. We have

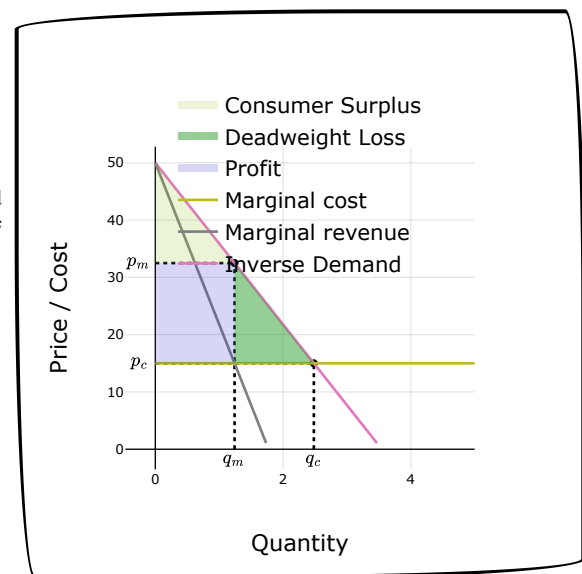
$$\pi_m = -\frac{(p_0 - c_1)^2}{4p_1}.$$

The consumer's surplus is

$$W_m^c = \frac{(p_0 - p_m)q_m}{2} = -\frac{(p_0 - c_1)^2}{8p_1}.$$

Therefore, the total welfare is

$$W_m = -\frac{3(p_0 - c_1)^2}{8p_1}$$



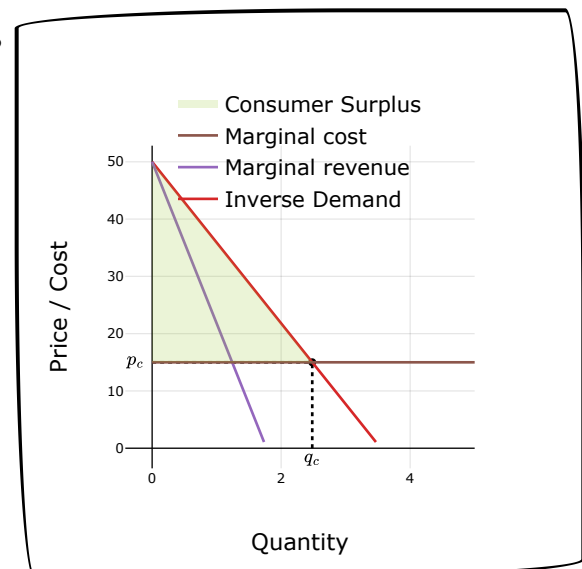
3. Perfect Competition

Profit is zero in perfect competition because the market price is equal to the firms' marginal cost. We have $p_e = c_1$, which implies that

$$q_e = -\frac{p_0 - c_1}{p_1}.$$

The total welfare is equal to the consumer's surplus

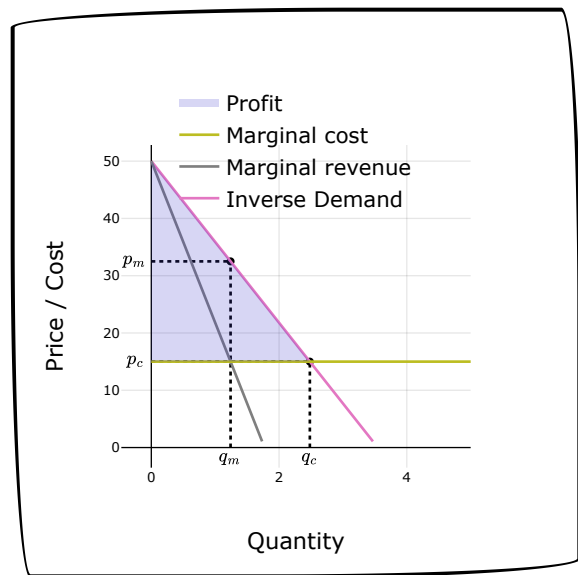
$$W_e = W_e^c = \frac{(p_0 - p_e)q_e}{2} = -\frac{(p_0 - c_1)^2}{2p_1}$$



4. Personalized pricing

In the case of personalized pricing, the situation is completely reversed. The consumer's surplus is zero, and the total welfare is equal to the profit of the firm, namely

$$W_p^s = \pi_p = W_c^c = -\frac{(p_0 - c_1)^2}{2p_1}$$



5. Comparison

	Monopoly	Competition	Personalized Pricing
Profit	$-\frac{(p_0 - c_1)^2}{4p_1}$	0	$-\frac{(p_0 - c_1)^2}{2p_1}$
Consumer Welfare	$-\frac{(p_0 - c_1)^2}{8p_1}$	$-\frac{(p_0 - c_1)^2}{2p_1}$	0
Total Welfare	$-\frac{3(p_0 - c_1)^2}{8p_1}$	$-\frac{(p_0 - c_1)^2}{2p_1}$	$-\frac{(p_0 - c_1)^2}{2p_1}$

9.14.2. Monopolistic Competition

Monopolistic competition is a structure describing a market with many firms offering similar, but not perfectly identical, commodities or services. Firms can affect prices but not as effectively as a monopolist.

1. A symmetric, affine demand, and liner cost example

Suppose $\bar{p}$ is the average price of the differentiated products. The firm's demand depends on the difference of its price from the average price. That is,

$$d(p) = q_0 \left(\frac{1}{n} + q_1(p - \bar{p}) \right) = \underbrace{\frac{q_0}{n} - q_0 q_1 \bar{p}}_{:= -\frac{p_0}{p_1}} + \underbrace{q_0 q_1 p}_{:= \frac{1}{p_1} p},$$

where $q_0, \bar{p} > 0, q_1 < 0$. The production costs of all firms have the form

$$c(q) = c_1 q,$$

with $c_1 > 0$.

2. Profit for a fixed number of firms in the market

For interior solutions, each firm sets

$$p = \frac{q_1 c - \frac{1}{n} + q_1 \bar{p}}{2q_1},$$

$$q = q_0 \frac{q_1 c + \frac{1}{n} - q_1 \bar{p}}{2}.$$

For symmetric solutions, these are simplified to

$$p = \frac{q_1 c - \frac{1}{n}}{q_1},$$

$$q = \frac{q_0}{n}.$$

3. What happens when the number of firms increases?

The market power of each individual firm decreases with n . The limiting case gives prices coinciding with the perfect competition price, namely $p \xrightarrow{n \rightarrow \infty} c$. Limiting quantities become irrelevant as the number of firms in the market increases, i.e., $q \xrightarrow{n \rightarrow \infty} 0$.

9.15. Exercises

9.15.1. Group A

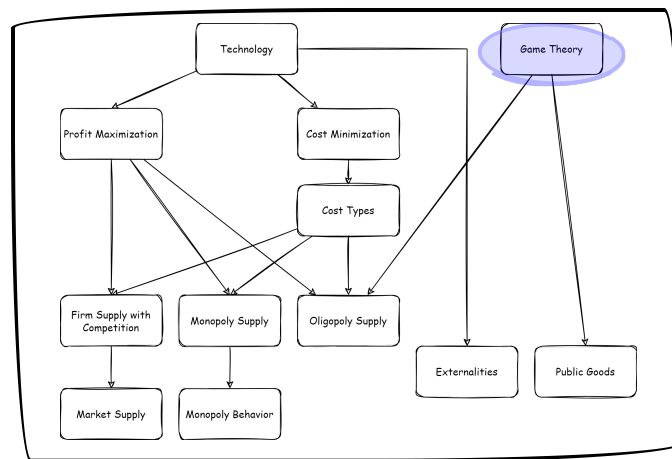
1. Suppose that a monopolist can adopt a group pricing strategy in a market with two groups. Both groups' demands have constant price elasticities correspondingly equal to $e_{d,1}$ and $e_{d,2}$. The marginal production cost is constant at c_1 , and there are no fixed cost. What price does the monopolist charge to each group?

10. Game Theory

10.1. Context

- In most real-life situations, economic agents do not operate in isolation. Their gains and losses depend not only on their own choices but also on the choices of others.
- Markets are typical examples of economic situations where social interactions matter.
 - How can we study social interactions in economics?
 - How do economic agents compete and coordinate with each other?
 - What are the social dilemmas that arise in such situations?

10.2. Course Structure Overview



10.3. Lecture Structure and Learning Objectives

Structure

- Street Fighter Mechanics (Case Study)
- Basic Concepts
- Examples
- A Spatial Competition Application
- A Market Entry Application
- Current Field Developments

Learning Objectives

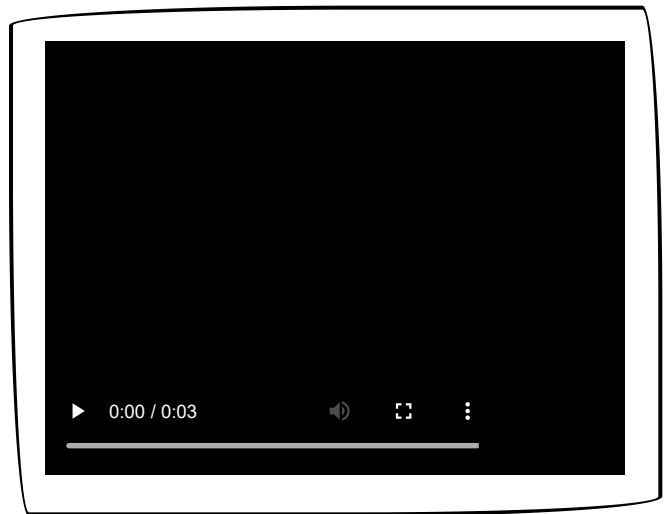
- Explain why social interactions can lead to social dilemmas.
- Explain how game theory models social interactions.
- Describe the concept of equilibrium in models with interactions.
- Illustrate the concept of equilibrium in static and dynamic market settings.

10.4. Street Fighter Mechanics

- Street Fighter II: The World Warrior is a fighting game released in 1991.
- It was originally released in arcade.
- It reestablished the arcade competition from high score chasing to one-vs-one play.
- It inspired competitive video game tournaments in the early 2000s.
- Today the e-sports market is valued at more than a billion dollars.
- Why was Street Fighter II so successful?

10.4.1. Command Grab

- Grapplers are (typically large) slow characters that have powerful grappling moves.
- The execution of these moves requires the avatars to be close.
- Command grabs are special grappling moves that cause a lot of damage.
- But they are very slow.



10.4.2. Neutral Jump

- Grabs do not work if the defender jumps vertically (neutral jump).
- Moreover, because command grabs are so slow, the defender can punish the grappler on his way down.



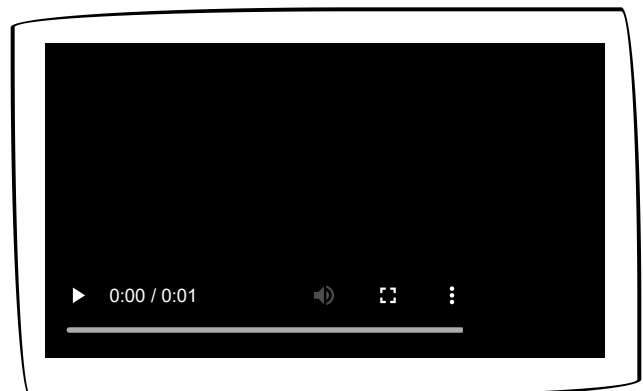
10.4.3. Normal Grab and Anti-Air

- Instead, the grappler can do a normal grab which recovers faster.
- In addition, with the fast recovery, the grappler can punish the defender in the air using a follow up, anti-air move.



10.4.4. Throw Technical

- An alternative option for the defender is to counter the grab with a technical counter.
- This tactic avoids the normal grab and anti-air punishment.
- However, it is vulnerable to the grappler's command grab.
- How do players resolve this situation?



10.5. Social Dilemmas

- For some social interactions, individual interests do not always work in favor of society as a whole.
- Individual producers' interests suggest using cheap, brown instead of more expensive, green technologies.
- However, if all producers act in this way, pollution increases and the lives of everyone become worse off.
- A **social dilemma** is a situation in which actions taken independently by agents pursuing their individual objectives result in inferior outcomes to other outcomes that are feasible if agents coordinate.

10.6. Social Interactions

- Game theory is the main apparatus used for examining social interactions.
- A **game** is a description of a social interaction specifying
 - the players (who is participating?),
 - the feasible actions (when is someone playing? What can she do?)
 - the information (what is known by players when making their decisions?)
 - the payoffs (what is the outcome for each possible combination of actions?)

10.6.1. Common Knowledge

- In the games that we will examine, the utilities and the choices of players are common knowledge.
- Common knowledge is information that is known and understood in the same way by all the players of a game.
- There is an element of infinite recursion in the idea of common knowledge.
- The agents of a game have **common knowledge** of a property **P** when they all know **P**, they all know that they know **P**, they all know that they all know that they know **P**, etc.

10.6.2. Actions and Strategies

- Each player in a game has one or more decisions to make.
- A single choice made at a particular decision node is called an **action**.
- The collection of all actions of a player in a game is called a **pure strategy** (or simply **strategy** when it is understood from context that it is pure).
- In games where a player has a single decision to make, her actions and strategies coincide.

10.7. Representations of Games

- Games can be represented in various ways.
- The representations are not always interchangeable.
- Some games admit only certain representations. Others can be represented in multiple ways.
- Each representation has certain advantages.

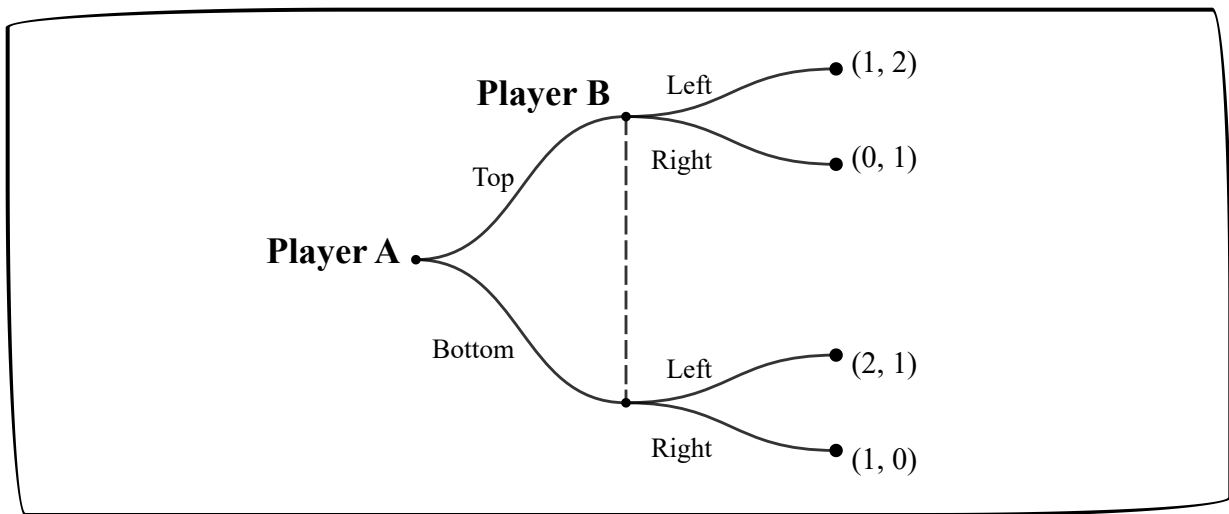
10.7.1. Normal Form

- Simple games can be represented using a table documenting the primitives of the game.
- This representation is called the **normal form** of a game.

		Player B	
		Left	Right
Player A	Top	1, 2	0, 1
	Bottom	2, 1	1, 0

10.7.2. Extensive Form

- The **extensive form** of a game is a representation in terms of a tree.
- The extensive form depicts more information than the normal form:
 - It illustrates the order in which the players act.
 - It illustrates the information available to each player.



10.7.3. Information Set

- If a player has the same information at two (or more) nodes, the nodes are connected with a dotted line.
- An **information set** is a collection of decision nodes that the player making decisions cannot distinguish at the time of decision.
- The player knows that she is located at one of the nodes of the information set, but she does not know at which one of them.

10.8. Best Responses

- Given a player's strategy, what is the best strategy with which the other player can respond?
- A **best response strategy** is a strategy that maximizes a player's payoff for given strategies of the remaining players of the game.
- The **best response mapping** is an association that gives the strategies that maximize a player's payoff for each combination of strategies of the remaining players of the game.

		Player B	
		Left	Right
Player A	Top	1, <u>2</u>	0, 1
	Bottom	<u>2</u> , <u>1</u>	<u>1</u> , 0

- $B_A(Left) = \{Bottom\}$, $B_A(Right) = \{Bottom\}$
- $B_B(Top) = \{Left\}$, $B_B(Bottom) = \{Left\}$

10.8.1. Dominant Strategies

- On some occasions, a player can choose a strategy that makes her better off irrespective of the strategies chosen by other players.
- A strategy that, for all strategies other players can choose, gives a higher payoff to a player compared to every other strategy available to her is called a **dominant strategy**.

		Player B	
		Left	Right
Player A	Top	1, <u>2</u>	0, 1
	Bottom	<u>2</u> , <u>1</u>	<u>1</u> , 0

- $Left \succ_B Right$

10.9. Nash Equilibria

- A collection of strategies, one for each player, such that each strategy constitutes a best response to the remaining players' strategies is called a **Nash equilibrium**.
- In short but less accurate, a Nash equilibrium is a collection of mutual best responses.
- Intuitively, a Nash equilibrium is a collection of strategies from which no one has an incentive to deviate.

		Player B	
		Left	Right
Player A	Top	1, <u>2</u>	0, 1
	Bottom	<u>2</u> , <u>1</u>	<u>1</u> , 0

- $NE = \{\{Bottom, Left\}\}$

10.9.1. Do Nash Equilibria Predict the Outcomes of Games?

- Nash equilibria do not say how, why, or whether these strategies are reached in a game.
- The definition of Nash equilibria suggests that if they are reached, then there is no incentive for anyone to change her behavior.

10.9.2. Are Nash Equilibria Unique?

- Nash equilibria are not unique.
- Multiple situations in a game may constitute points from which no one wants to deviate.
- The payoffs of the players in different Nash equilibria can be significantly different.

		Player B	
		Left	Right
Player A	Top	1, 1	<u>1</u> , <u>2</u>
	Bottom	<u>2</u> , <u>1</u>	0, 0

- $NE = \{\{Bottom, Left\}, \{Top, Right\}\}$

10.9.3. Are Nash Equilibria Necessarily Pareto Efficient?

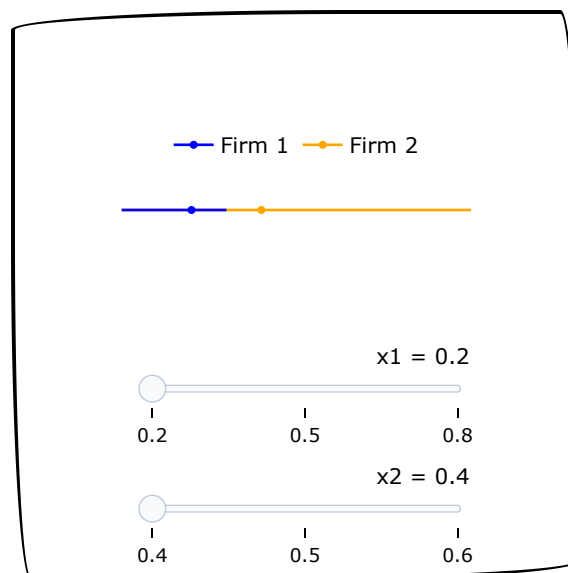
- No. Nash equilibria can be Pareto inefficient.
- A classic example is the prisoner's dilemma.

		Suspect B	
		Confess	Deny
Suspect A	Confess	<u>-3</u> , <u>-3</u>	0, -6
	Deny	-6, <u>0</u>	-1, -1

10.10. Spatial Competition

- There are two firms on a street.
- Points on the street are given by $[0, 1]$.
- Each firm chooses a point.
- Firms have the same cost and charge the same price.
- Customers on the street prefer the firm that is the closest.

10.10.1. An Illustration of the Game



10.10.2. Non Equilibrium Placements

- If firm 2 chooses $x_2 > \frac{1}{2}$, firm 1 would like to undercut by a small amount and set $x_1 = x_2 - \epsilon > \frac{1}{2}$.
- Then firm 2 has a profitable deviation by changing to $x_2 = \frac{1}{2}$.
- Thus any $x_2 > \frac{1}{2}$ cannot be an equilibrium.
- Similarly, any $x_2 < \frac{1}{2}$ cannot be an equilibrium.
- Analogous arguments hold for firm 1 because of symmetry.

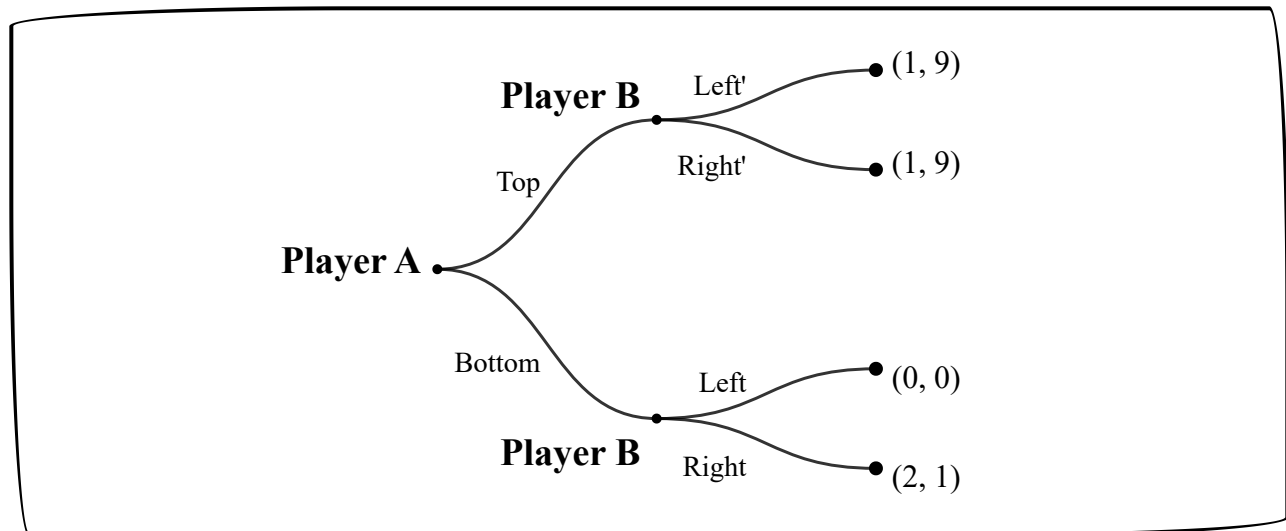
10.10.3. Equilibrium Placements

- Therefore, the only possible equilibrium is $x_1 = \frac{1}{2} = x_2$.
- Firms split the market and make equal profits.
- Any deviation leads to less profit for the firm that moved.

10.11. Sequential Games

- A game is called **sequential** if its players play sequentially instead of simultaneously.
- Nash equilibria also exist in such games.
- We can find some of them (the subgame-perfect ones) using backward induction.
 - The best action of the player that acts at the last date is calculated.
 - Given this best response, the best action of the player that acts at the previous to last date is calculated.
 - We continue in this fashion until we have calculated the best action of the player who acts at the initial date.

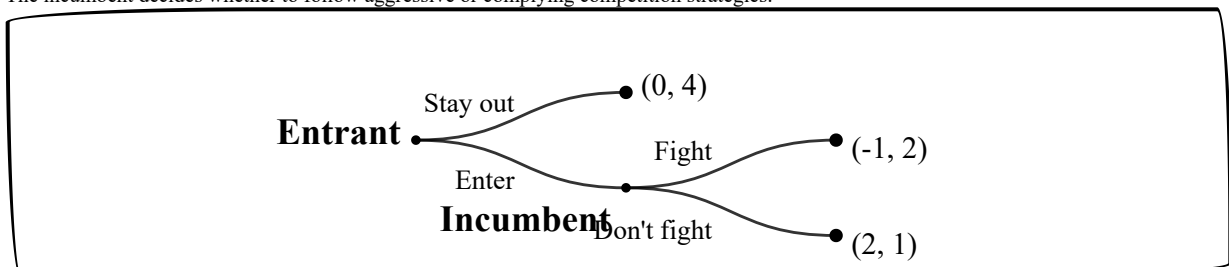
10.11.1. Backward Induction



- $NE = \{\{Bottom, (Left', Right)\}, \{Bottom, (Right', Right)\}\}$

10.12. A Game of Market Entry

- Consider a market with one firm already operating and a potential entrant.
- The entrant decides whether to enter the market.
- The incumbent decides whether to follow aggressive or complying competition strategies.



- $NE = \{\{Stay out, Fight\}\}$

10.13. Current Field Developments

- Game theory has a deep theoretical basis and a wide variety of applications.
- In the last 60 years, it grew to be one of the most active research areas in many sciences.
- Current game theory models in economics involve behavioral biases and cognitive limitations.
- Game theory is also used in political sciences to analyze topics ranging from voters' behavior to war conflicts.
- Computer science uses game theory concepts and tools to develop artificial intelligence agents.
- In biology, game theory has been used to describe some social aspects of evolutionary processes.

10.14. Concise Summary

- Social dilemmas are situations in which private actions can lead to inferior social outcomes.
- Social interactions can be analyzed using game theory.
- Social interaction settings can be either static or dynamic.
- Equilibria in social interaction settings are situations in which no one has an incentive to change her behavior.
- Such equilibria are not necessarily economically efficient.

10.15. Further Reading

- Varian (2010, secs. 29.1-6)
- CORE Team (2017, secs. 4.1-5, 4.13)
- Nash (1950)

10.16. Mathematical Details

10.16.1. Mixed Strategies

Pure strategies are collections of actions, one for each decision to be made. Sometimes the players prefer to choose strategies based on some randomization rule. Players can randomize by assigning the probabilities (weights) with which they use their pure strategies. A distribution over the player's pure strategies is called a mixed strategy.

For example, suppose that player **A** has two pure strategies (**Top** and **Bottom**). The mixed strategy $(p, 1 - p)$ assigns probability p to choosing **Top** and probability $1 - p$ to choosing **Bottom**.

Finite games always have at least one Nash equilibrium in mixed strategies (Nash 1950). A game is finite if the numbers of players, actions, and decision nodes are finite.

10.16.2. The Grappler Game

Going back to the Street fighter mechanics, suppose that the payoffs in the interaction between a grappler and a defender are given by the following table.

		Defender	
		Neutral Jump	Tech
Grappler	Command Grab	-2, <u>2</u>	<u>3</u> , -3
	Normal Grab	<u>2</u> , -2	0, <u>0</u>

How can we calculate the mixed strategy Nash equilibrium in this case? The grappler has two pure strategies (**Command Grab** and **Normal Grab**). The mixed strategy $(p, 1 - p)$ assigns probability p to choosing **Command Grab** and probability $1 - p$ to choosing **Normal Grab**. The grappler chooses these probabilities so that it makes the defender indifferent between **Neutral Jump** and **Tech** (why?).

$$\underbrace{2p + (-2)(1 - p)}_{\text{Expected payoff when the defender chooses Neutral Jump}} = \underbrace{(-3)p + 0(1 - p)}_{\text{Expected payoff when the defender chooses Tech}} \implies p = \frac{2}{7}.$$

Similarly the defender mixes **Neutral Jump** with q and **Tech** with $1 - q$, such that

$$\underbrace{(-2)q + 3(1 - q)}_{\text{Expected payoff when the grappler chooses Command Grab}} = \underbrace{2q + 0(1 - q)}_{\text{Expected payoff when the grappler chooses Normal Grab}} \implies q = \frac{3}{7}.$$

The Nash equilibrium of the grappler game is

$$\left\{ \left(\frac{2}{7}, \frac{5}{7} \right), \left(\frac{3}{7}, \frac{4}{7} \right) \right\}.$$

10.16.3. Rock Paper Scissors

There is no pure strategy Nash equilibrium. However, there is a symmetric mixed strategy equilibrium in which both players randomize using $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})$.

		Player B		
		Rock	Paper	Scissors
Player A	Rock	0, 0	-1, <u>1</u>	<u>1</u> , -1
	Paper	<u>1</u> , -1	0, 0	-1, <u>1</u>
	Scissors	-1, <u>1</u>	<u>1</u> , -1	0, 0

10.16.4. Repeated Games

Repeated games are games played at a finite or infinite number of dates, and at each date, the same strategic interaction is repeated. The strategic interaction repeated at each date is called a stage game. When there is a future, threats can be used to sustain cooperation.

For example, suppose that the prisoner's dilemma is used as the stage game of a repeated game with an infinite time horizon. The payoff of all future dates is discounted by $0 < \delta < 1$. Is there any room for coordination in this case? Players can coordinate by using trigger strategies: If at all previous dates the other player has denied, then deny. Otherwise, confess. If players coordinate, then their payoffs are

$$u^{c,i} = -\frac{1}{1-\delta}$$

If player i deviates at the current date, then her payoff is

$$u^{d,i} = -3\frac{\delta}{1-\delta}$$

Coordination can be supported if

$$u^{c,i} \geq u^{d,i} \iff \delta \geq \frac{1}{3}$$

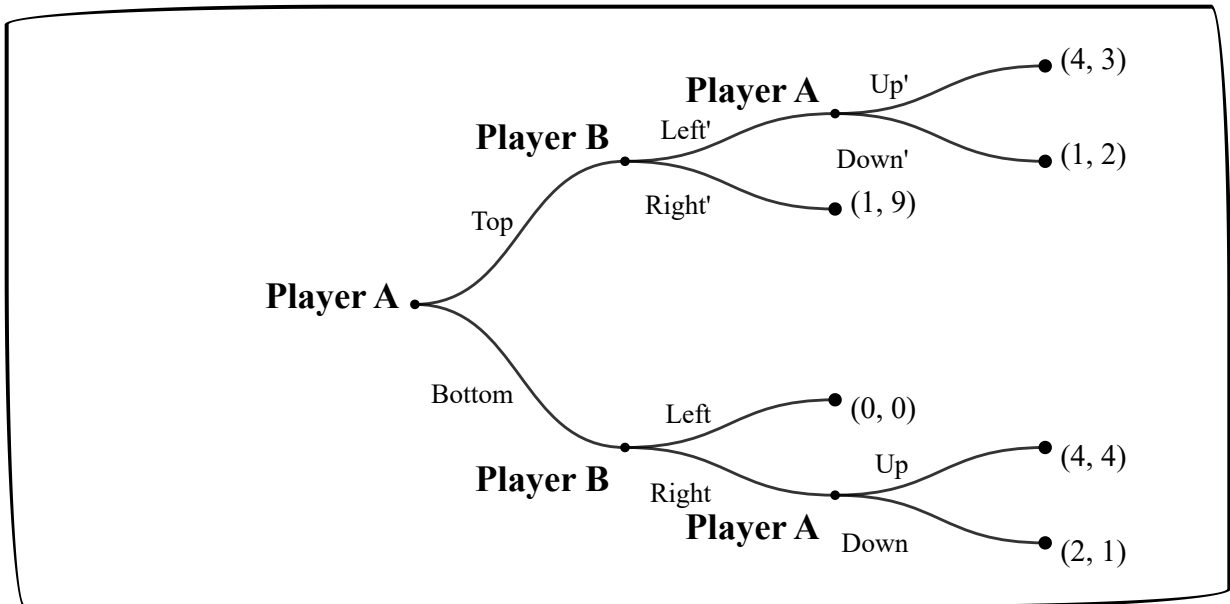
10.17. Exercises

10.17.1. Group A

1. Consider the following game.

		Player B	
		Foo	Bar
Player A	Foo	2, 1	0, 0
	Bar	0, 0	1, 2

- a. Find all the pure strategy Nash equilibria.
 - b. How do you interpret the objective of the players in this game?
2. Suppose that two players play a game, and it is known with certainty that player **A** is not choosing a Nash equilibrium strategy of a game. Should player **B** choose her Nash equilibrium strategy?
 3. Find the equilibrium of the following game using backward induction.



4. Consider the game

		Player B	
		Left	Right
Player A	Top	3, 3	0, 1
	Bottom	1, 0	2, 2

- a. Find all the pure strategy Nash equilibria of the game.
- b. Suppose that player A plays first. Draw the extensive form of the game.
- c. Solve the sequential game using backward induction.
- d. Are all Nash equilibria of the static game present in the sequential game? Why, or why not?

10.17.2. Group B

1. Consider the game

		Player B	
		Left	Right
Player A	Top	0, 2	2, 0
	Bottom	2, 0	0, 2

- Does the game have any pure strategy Nash equilibria?
- Find all the Nash equilibria of the game.

2. We consider the prisoner's dilemma as the stage game of a repeated game.

- Suppose that the stage game is repeated twice. Find the unique Nash equilibrium of the game.
- Suppose that the stage game is repeated an arbitrary finite number of times (say $n = 10^{10}$). Find the unique Nash equilibrium of the game.
- Why is it impossible for the players to coordinate and play the Pareto efficient strategies in the above cases?

10.17.3. Group C

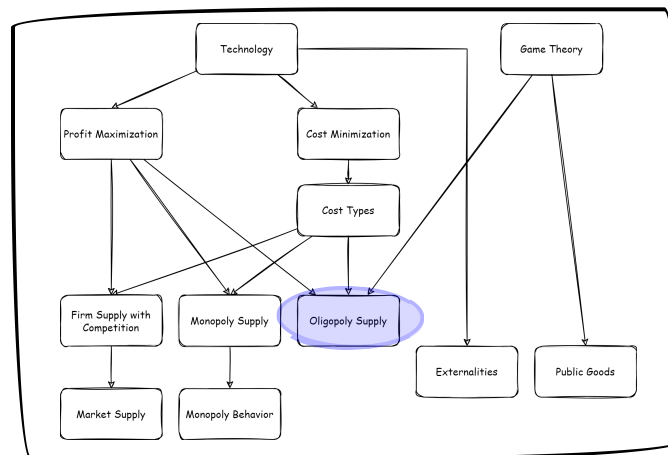
- Consider the prisoner's dilemma game in which if both players do not confess, they get a payoff of -1 ; if they both confess, they get a payoff of -6 ; and if one confesses and the other does not, the one that confessed receives a payoff of 0 and the other a payoff of -10
 - Classify the strategy profiles into those yielding Pareto efficient and Pareto inefficient allocations.
 - Show that for each strategy profile yielding a Pareto efficient allocation, there exist positive weights $(\alpha, 1 - \alpha)$ such that the weighted average of the payoffs of the players for this profile is greater or equal to the weighted average payoffs of all other strategy profiles.
 - Show that for each strategy profile yielding Pareto inefficient allocations, such weights do not exist.

11. Oligopoly

11.1. Context

- Many real markets are neither perfectly competitive nor monopolies. Instead, they are oligopolies comprising a small number of firms that have large enough market shares and can influence prices.
- Nonetheless, firms' profits do not exclusively depend on their own choices. Their small numbers allow them to utilize a variety of cooperation and competition strategies.
 - How do firms strategically interact?
 - What means do they use to compete?
 - How do the welfare outcomes of oligopolies compare with those of monopolies and perfect competition?

11.2. Course Structure Overview



11.3. Lecture Structure and Learning Objectives

Structure

- Our Customers are our Enemies (Case Study)
- Basic Concepts
- The Cournot and Bertrand models
- Examples of the two models
- Welfare Comparisons
- Current Field Developments

Learning Objectives

- Explain how game theory models oligopolistic competition.
- Describe oligopolies with competition in quantities and their welfare output.
- Describe oligopolies with competition in prices and their welfare output.
- Illustrate the welfare differences between oligopolies, monopolies, and perfect competition.

11.4. Our Customers are Our Enemies

- Lysine is an amino acid that speeds the development of lean muscle tissue in humans and animals.
- It is essential for humans, but we cannot synthesize it.
- It has to be obtained from the diet.

11.4.1. The Lysine Industry

- At the end of the 1980s, the world lysine industry consisted of three significant sellers:
 - Ajinomoto,
 - Kyowa, and
 - Sewon.
- The three largest consumption regions were Japan, Europe, and North America.
- Most production took place in Japan, but it was based on imports of US dextrose.
- Ajinomoto had the largest share of the world market.

11.4.2. The ADM Entry

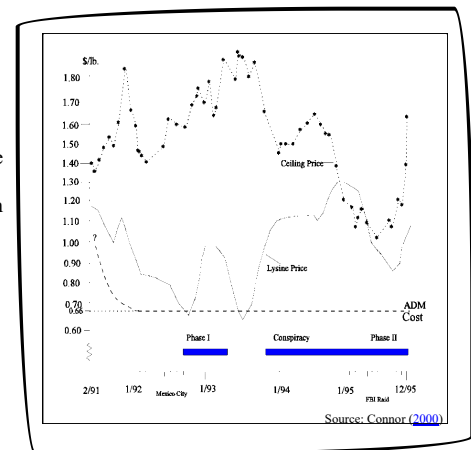
- In February 1991, Archer Daniel Midland Co. (ADM) entered the market and built by far the world's largest lysine plant in the US.
- ADM hired biochemist Mark Whitacre, Ph.D., as head of the new division.
- ADM's plant was three times the size of Ajinomoto's largest plant.
- ADM gave Ajinomoto and Kyowa executives an unrestricted tour to show its production capacity.
- Companies engaged in a price war.
 - Three months before ADM's entry, the average US lysine price was **\$1.22** per pound.
 - After an 18-month price war, the US price averaged **\$0.68** per pound.
 - ADM's share of the US market reached **80%**.

11.4.3. The Lysine Association

- After the price war, ADM was willing to soften competition.
- In 1992, Mark Whitacre and his boss Terrance Wilson met with top Ajinomoto and Kyowa managers.
- Wilson proposed forming a world lysine association that would regularly meet.
- The new association would collect and distribute market information.
- Wilson also suggested that the new association could provide a convenient cover for illegal price-fixing discussions! ([Connor 2001](#))
- After a year, the lysine association was founded, met quarterly, and performed the two functions that Wilson proposed.

11.4.4. Price Fixing

- There were **25** price fixing meetings in total.
- The first one took place in the Nikko Hotel in Mexico on June 23, 1992.
- The average Lysine price immediately jumped by more than **12%**.
- Consensus was not always easy to reach. The companies distrusted each other!
- There was a breakdown of the cartel during the spring and summer of 1993, and the lysine price plummeted.
- The crisis was resolved at a meeting in Irvine, California in October 1993 between ADM's and Ajinomoto Executives. But...
- This meeting and many others were caught on video by the FBI.



11.4.5. Frenemies

WILSON: The only thing we need to talk here because we are gonna get manipulated by these God damn buyers, they're sh, they can be smarter than us if we let them be smarter.

MIMOTO: (Laughs).

WILSON: Okay?

MIMOTO: (ui).

WILSON: They are not your friend. They are not my friend. And we gotta have 'em. Thank God we gotta have 'em, but they are not my friends. You're my friend. I wanna be closer to you than I am to any customer. 'Cause you can make us, I can make money, I can't make money. At least in this kind of a market. And all I wanna is ta tell you again is let's-let's put the prices on the board.

[Co-conspirator explains how end-of-year compensation scheme eliminates incentives to cheat on cartel](#)

11.4.6. The Whistle-blower

- In 1992, Mark Whitacre became an FBI whistle-blower. He is the highest-level corporate executive to ever have done so.
- Whitacre's wife pressured him into becoming a whistle-blower. She threatened to inform the FBI herself if Whitacre wouldn't do it.
- Whitacre informed the FBI that he and other ADM executives performed illegal price-fixing operations.
- Over the next three years, Whitacre collected information and recorded conversations with ADM executives and competitors.
- The ADM settlement involved the greatest US federal charges at the time.
- Whitacre was convicted for embezzling and money laundering, which he performed while cooperating with the FBI in the price fixing case.

11.4.7. What can we Learn?

- Why did ADM initially engage in a price war?
- Why did it initiate the price fixing discussions afterward?
- Why was there so much distrust among companies?

11.5. Competition and Cooperation

- **Oligopoly** refers to market structures with a small number of interdependent firms.
 - Oligopolistic firms typically compete using non-cooperative strategies.
 - On some occasions, firms collude and use cooperative strategies.

11.5.1. Non-cooperative Overview

- Oligopolies may compete using pricing strategies or by choosing quantities.
- Different means of competition strategies crucially affect the market outcome.
- The means of competition are decisive components of the market structure.

11.5.2. Cooperation and Collusion

- Oligopolies have used explicitly collusive strategies in the past (e.g., cartels).
- Nowadays, collusion is usually illegal.
- Instances of tacit collusion have also been documented.
- Tacit collusion strategies do not explicitly require the exchange of information.

11.6. Competition in Quantities

- The **Cournot** model of oligopoly describes a market structure with two or more firms such that
 - the market does not suffer from any other market failure (imperfect information, externalities, etc.),
 - no other firms can enter the market,
 - firms sell a homogeneous product,
 - firms try to maximize their profits,
 - consumers are price takers; firms are simultaneously choosing the quantities that they produce, and
 - consumers try to maximize their utility.

11.7. A Cournot Competition Example

- Suppose that there are two firms $(i, j \in \{1, 2\})$ in the market.
- They have the cost functions

$$c(q_i) = 4q_i.$$

- Let the market inverse demand be

$$p(q_i + q_j) = 28 - 2(q_i + q_j).$$

- Each firm maximizes its profit

$$\max_{q_i} \{(28 - 2(q_i + q_j))q_i - 4q_i\}.$$

11.7.1. Best Responses

- The necessary condition for each firm is

$$28 - 2q_j - 4q_i = 4$$

- Solving for q_i gives the best response of firm i

$$q_i = \frac{24 - 2q_j}{4}.$$

11.7.2. Nash Equilibrium

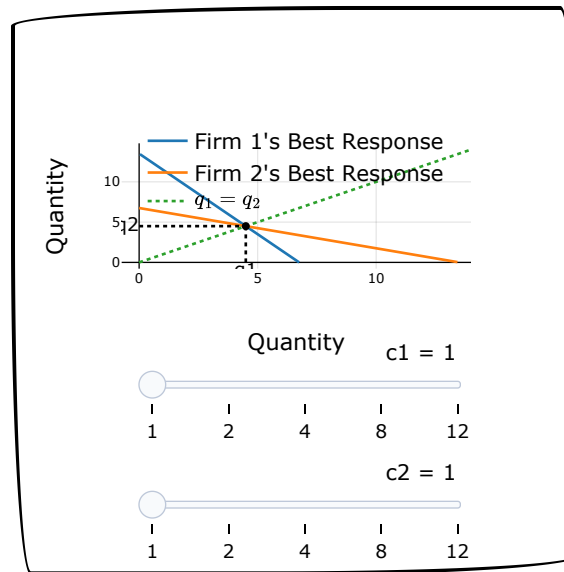
- Combining the two best responses gives the Nash equilibrium

$$q_i = 4 = q_j.$$

- Profits are then

$$\pi_i = 32.$$

11.7.3. What Happens if Costs are Not Symmetric?



11.8. Competition in Prices

- The **Bertrand** model of oligopoly describes a market structure with two or more firms such that
 - the market does not suffer from any other market failure (imperfect information, externalities, etc.),
 - no other firms can enter the market,
 - firms sell a homogeneous product,
 - firms try to maximize their profits,
 - consumers are price takers, and firms are simultaneously choosing prices, and
 - consumers try to maximize their utility.

11.9. A Bertrand Competition Example

- Suppose that there are two firms ($i, j \in \{1, 2\}$) in the market.
- Their marginal costs are equal to 4.
- Let the demand for firm i be

$$d_i(p_i, p_j) = \begin{cases} 10 - \frac{1}{2}p_i & p_i < p_j \\ 5 - \frac{1}{4}p_i & p_i = p_j \\ 0 & p_i > p_j \end{cases}$$

- The firm with the lowest price gets all the demand.
- If prices are equal, demand is equally split.

11.9.1. Non-Equilibrium Prices

- Suppose that firm j sets a price p_j that is greater than the marginal cost of firm i (i.e., 4).
- Firm i can undercut by a small amount and grab all the market. For instance, set price $p_i = \frac{p_j + 4}{2}$.
- Thus, firm i can only set a price equal to firm j 's marginal cost.
- Analogous arguments hold for firm j 's strategy.

11.9.2. Equilibrium

- The only possible equilibrium is to set a price equal to the (common) marginal cost.
- Firms do not have any incentive to deviate.
- Setting lower prices leads to losses.
- Setting higher prices leads to zero profit.
- Even with two firms, price competition leads to price setting similar to the competitive equilibrium case.

11.10. Current Field Developments

- There are two main types of extensions of the basic models (Cournot and Bertrand),
 - extensions incorporating dynamic decisions (e.g., Stackelberg), and
 - extensions incorporating dynamics under uncertainty.
- Oligopoly models are primarily used in industrial organization (see Belleflamme and Peitz (2010) for an introduction) to examine
 - Market power
 - Pricing strategies,
 - Competition policies, and

- R&D and innovation.
- Some recent micro-founded, general equilibrium macro models describe frictions with oligopolistic markets.

11.11. Concise Summary

- Competition is not always perfect.
- In reality, a few large firms have the lion's share in many markets.
- Such markets are described by oligopoly models.
- Oligopolies can compete or collude. Explicit collusion is illegal in the US and EU.
- Depending on how firms compete (prices or quantities) and the number of firms, the oligopoly model gives predictions with welfare properties that range from perfect competition to monopoly.

11.12. Further Reading

- Varian ([2010, chap. 28](#))
- Belleflamme and Peitz ([2010, secs. 3.1.1, 3.2](#))
- Connor ([2001](#))

11.13. Mathematical Details

11.13.1. Simultaneous Quantity Competition with two Firms

Firms choose their strategies at the same time. Both firms choose their supplied quantities. This market structure is known as the Cournot model of competition.

1. The problem

Each firm solves

$$\max_{q_i} \{p(q_1 + q_2)q_i - c(q_i)\}.$$

2. Best responses

The necessary condition for each firm is

$$p'(q_1 + q_2)q_i + p(q_1 + q_2) = c'(q_i).$$

From these conditions, the two best responses are obtained

$$q_i = b_i(q_j) \quad (i \neq j).$$

Solving the system of these two equations gives the equilibrium point (if it exists).

3. An affine demand and symmetric, linear costs example

For inverse demand and costs given by

$$\begin{aligned} p(q) &= p_0 + p_1 q \\ c(q) &= c_1 q, \end{aligned}$$

the best responses become

$$q_i = \frac{c_1 - p_0 - p_1 q_j}{2p_1}.$$

The equilibrium quantities are given by

$$q_i = \frac{c_1 - p_0}{3p_1}.$$

The profits are symmetric and can be calculated as

$$\pi_i = -\frac{(c_1 - p_0)^2}{9p_1}.$$

4. An affine demand and non-symmetric, linear costs example

What happens if costs are not symmetric in the affine example? Suppose demand is as before and costs are

$$c_i(q) = c_{1,i}q, \quad (i = \{1, 2\})$$

The best responses become

$$q_i = \frac{c_{1,i} - p_0 - p_1 q_j}{2p_1}.$$

Equilibrium ceases to be symmetric. The equilibrium quantities are given by

$$q_i = \frac{2c_{1,i} - c_{1,j} - p_0}{3p_1}.$$

The symmetric equilibrium quantities are obtained as a special case from the last formula by setting $c_{1,i} = c_{2,j}$.

Firm i produces more than j if and only

$$\frac{2c_{1,i} - c_{1,j} - p_0}{3p_1} \geq \frac{2c_{1,j} - c_{1,i} - p_0}{3p_1},$$

which, because $p_1 < 0$, is equivalent to $c_{1,i} \leq c_{1,j}$. Thus, the lower-cost firm produces more.

The total market quantity is

$$q = \frac{2c_{1,i} - c_{1,j} - p_0}{3p_1} + \frac{2c_{1,j} - c_{1,i} - p_0}{3p_1} = \frac{c_{1,i} + c_{1,j} - 2p_0}{3p_1},$$

and the market price

$$p(q) = \frac{c_{1,i} + c_{1,j} + p_0}{3}.$$

We can then calculate the profit of firm i as

$$\begin{aligned} \pi_i &= \left(\frac{c_{1,i} + c_{1,j} + p_0}{3} - c_{1,i} \right) \frac{2c_{1,i} - c_{1,j} - p_0}{3p_1} \\ &= -\frac{(2c_{1,i} - c_{1,j} - p_0)^2}{9p_1}. \end{aligned}$$

The firm that produces more makes the greatest profit. The easiest way to get this result is to rewrite profits as $\pi_i = -q_i^2 p_1$. Since $p_1 < 0$, we have $\pi_i \geq \pi_j$ if and only if $q_i \geq q_j$.

11.13.2. Simultaneous Quantity Competition with More Than two Firms

We extend the problem by allowing $n > 2$ firms that simultaneously choose their strategies. All firms choose their supplied quantities.

How is equilibrium affected when more than two firms are in the market? Each firm solves

$$\max_{q_i} \left\{ p \left(\sum_{j=1}^n q_j \right) q_i - c(q_i) \right\}.$$

Analogously to the two-firm case, we obtain n best response functions

$$q_i = b_i((q_j)_{j \neq i}) \quad (i = 1, \dots, n).$$

Solutions to the system of best responses (if any) are the Nash equilibrium of this oligopoly model.

1. An affine demand and symmetric, linear costs example

The best responses become

$$q_i = \frac{c_1 - p_0 - p_1 \sum_{j \neq i} q_j}{2p_1}.$$

The equilibrium quantities are given by

$$q_i = \frac{c_1 - p_0}{(n+1)p_1}.$$

Profits are then

$$\pi_i = -\frac{(c_1 - p_0)^2}{(n+1)^2 p_1}.$$

The total quantity is

$$q = \sum_{i=1}^n q_i = \sum_{i=1}^n \frac{c_1 - p_0}{(n+1)p_1} = \frac{n}{n+1} \frac{c_1 - p_0}{p_1},$$

so the market price becomes

$$p(q) = p_0 + p_1 n \frac{c_1 - p_0}{(n+1)p_1} = \frac{nc_1 + p_0}{n+1}.$$

The case of two firms can be obtained by replacing $n = 2$ in the above results. We can also obtain the solution to the monopoly problem if we set $n = 1$.

Profits decrease as the number of firms in the market increases, with the limiting case being

$$\pi_i \xrightarrow{n \rightarrow \infty} 0.$$

In addition, we have $p(q) \rightarrow c_1$, $q_i \rightarrow 0$, and $q \rightarrow \frac{c_1 - p_0}{p_1}$ as $n \rightarrow \infty$. The production of each firm becomes negligible, and the total market quantity and price approach those of perfect competition.

11.14. Exercises

11.14.1. Group A

1. Suppose that there are two firms in a market with an affine inverse demand function $p(q) = p_0 + p_1 q$, where $p_0 > 0$ and $p_1 < 0$. The firms compete by simultaneously choosing quantities and have constant marginal costs equal to c_1 . Production has no fixed cost. Find the equilibrium outputs, price, and profits.
2. Can an oligopolistic market structure result in an efficient level of output?
3. Consider a market with inverse demand function $p(q) = 100 - 2q$. The total cost function for any firm in the market is given by $c(q) = 4q$.
 - a. What is the marginal cost for any firm in the market?
 - b. Calculate the perfect competition market quantity and price.
 - c. Suppose that the market consists of two firms that compete by simultaneously choosing their supplied quantities. Find the best responses and the equilibrium market output and price?
 - d. Draw the two best responses in a single graph to indicate the equilibrium point.
 - e. Suppose that the two firms collude by maximizing and splitting their joint profit. Find market output and the market price. Compare them with the non-collusive market output and price.
 - f. Suppose that a firm decides to deviate from the collusive output, while the other one keeps its collusive strategy unchanged. What are the firms' output levels and profits?
 - g. Is the collusion strategy sustainable in this game?

11.14.2. Group B

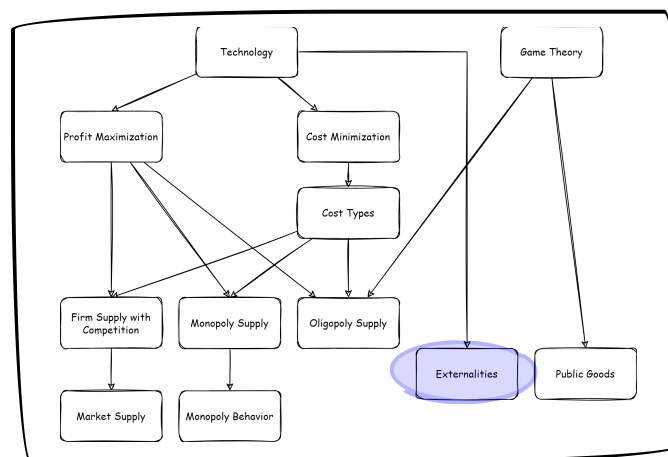
1. Give an example in which there is no equilibrium in a duopoly market in which firms simultaneously compete using quantities. Draw the best responses of the firms in this market.
2. Suppose there are n identical firms in a market competing by simultaneously choosing quantities. Show that the elasticity of the market demand must be less than $-1/n$ for an equilibrium to exist.

12. Externalities

12.1. Context

- Free markets of regulation? Ideal free markets result in economically efficient allocations. Property rights and enforceable contracts are two necessary conditions for a free market structure to be viable.
- Nevertheless, property rights and enforceable contracts are not always available and the presence of externalities can lead to market failures in such situations.
 - What do economists mean when they talk about externalities?
 - What are the welfare implications of such externalities?
 - Can interventions improve the welfare conditions of the agents?

12.2. Course Structure Overview



12.3. Lecture Structure and Learning Objectives

Structure

- The Aralkum Desert (Case Study)
- Basic Concepts
- An externality Example
- Application: Predictions in the Aral Sea case
- Welfare Analysis and Improvements.
- Current Field Developments

Learning Objectives

- Illustrate production externalities using an ecological collapse case study.

- Explain the various types of externalities.
- Describe the ecological collapse by an externality model.
- Compare model predictions with the case study events.
- Explain how economic efficiency can be obtained in the presence of externalities.

12.4. The Aralkum Desert

- The Aralkum desert is located in central Asia, shared between Uzbekistan and Kazakhstan.
- It is the newest desert on the planet.
- It appeared in **1960**.



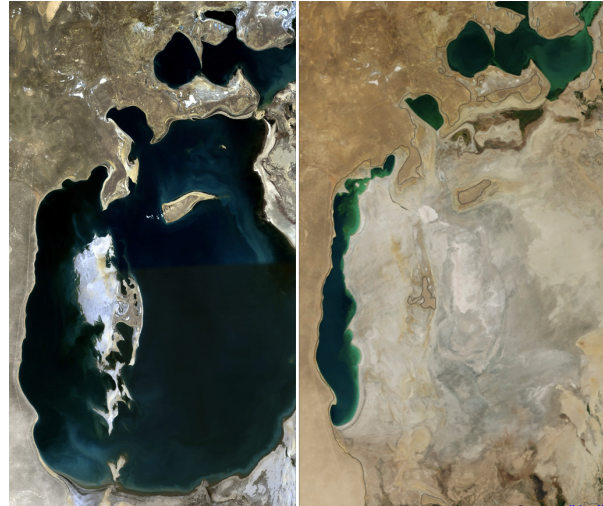
12.4.1. The Desert of Forgotten Ships

- It is remarkable in another peculiar way.
- It is the desert with the most boats and ships!



12.4.2. The Aral Sea Aralkum Desert

- It used to be the fourth largest lake on the planet, called the Aral Sea.
- In 1997, it was **10%** of its original size.
- By 2014, its eastern basin had completely dried up.
- It is now called the Aralkum desert.



Source: [wikipedia](https://www.wikipedia.org)

12.4.3. The White Gold

- In the early 1960s, the Soviet government planned for cotton, or 'white gold', to become a major exporting industry.
- Amu Darya and Syr Darya rivers were diverted from feeding the Aral Sea to irrigating the desert.
- The amount of water taken from the rivers doubled between 1960 and 2000.
- The plan was a success.
- In 1988, Uzbekistan was the largest exporter of cotton in the world.
- In 2006, **17%** of Uzbekistan's exports came from cotton.

12.4.4. The Ship Graveyards

- The Aral Sea used to have a thriving fishing industry.
- It sustained around **40** thousand professional fishermen.
- It produced about **17%** of the Soviet Union's fish catch.
- The salinity of the remaining lake became too high for 20 native fish species to survive.
- Cities with harbors were deserted and became ship graveyards.
- The local populations have dramatically declined.



Source: [wikipedia](https://www.wikipedia.org)

12.5. Production Externalities

- On some occasions, agents' actions outside of a production process affect the outcome of production.
- Environmental spillover cases are classic examples of negative effects.
 - E.g., a fishery and any number of consumer/producer polluters placed on the same river.
- The impact of external agents does not need to be negative.
 - E.g., an apple orchard and a beekeeper placed next to each other.
- A production technology exhibits a **production externality** when its production set (or production output) depends on the choices of economic agents other than the producing firm.

12.5.1. Types of Production Externalities

- A **positive production externality** is an externality for which a production process is boosted by the actions of external economic agents that are not beneficiaries of the profit derived from the process.
- A **negative production externality** is an externality for which a production process is hindered by actions of economic agents that are not beneficiaries of the profit derived from the process.

12.6. Consumption Externalities

- We are focusing on production, but similar concepts are applicable on the consumption side of the economy (e.g., the decision problem studied in Microeconomics I).
 - Listening to loud music at 3:00 AM can (negatively) affect your neighbors.
 - Maintaining a beautiful garden can (positively) affect your neighbors.
- A commodity or a service exhibits a **consumption externality** when its consumption from one agent affects the utility of other agents.

12.6.1. Types of Consumption Externalities

- A **positive consumption externality** is an externality for which the utility of other agents is positively affected.
- A **negative consumption externality** is an externality for which the utility of other agents is negatively affected.

12.7. A Pollution Externality Example

- Suppose there are two markets with one profit maximizing firm in each of them.
- The firm of market **1** produces cotton and uses a river for irrigation. The more water it draws from the river, the lower the production cost becomes.
- The firm of market **2** is a fishery. It does not control the level of water in the river. However, the less water there is in the river, the more costly it is to catch fish.
- Let $p_c = 5$ be the price of cotton and $p_f = 2$ be the price of fish. The water level is not priced in the market.
- Let $\xi \in [0, 1]$ be a variable that measures how much water out of the river is used for irrigation as a percentage of its full capacity.

12.7.1. Costs with Externalities

- The cotton producer has production cost

$$c_c(q_c, \xi) = q_c^2 (5 + (1 - \xi)^2)$$

- The cost is decreasing in the percentage of water used in irrigation.
- The fishery cost function is

$$c_f(q_f, \xi) = q_f^2 (1 + \xi^2)$$

- The cost is increasing in the percentage of water used in irrigation (externality).

12.7.2. The Firms' Problems

- The cotton producer solves

$$\max_{q_c, \xi} \{ p_c q_c - q_c^2 (5 + (1 - \xi)^2) \}$$

- The fishery problem is

$$\max_{q_f} \{ p_f q_f - q_f^2 (1 + \xi^2) \}.$$

- The fishery does not control the level of externality, but its cost function is increasing in it.

12.7.3. The Market Solution

- Since $\frac{\partial c_c(q_c, \xi)}{\partial \xi} < 0$, the cotton producer chooses $\xi = 1$.
- Moreover, the cotton producer would like to choose q_c such that

$$5 = 2q_c (5 + (1 - \xi)^2)$$

- This gives $q_c = \frac{1}{2}$.
- The fishery chooses

$$2 = 2q_f (1 + \xi^2)$$

- This gives (for $\xi = 1$) $q_f = \frac{1}{2}$.

12.7.4. The Market Solution's Welfare

- The river is dried out ($\xi = 1$).
- The cotton producer has profit

$$\pi_c = 5 \frac{1}{2} - \frac{1}{4} 5 = \frac{5}{4}$$

- The fishery has profit

$$\pi_f = 2 \frac{1}{2} - \frac{1}{4} 2 = \frac{1}{2}$$

12.8. The Merged Firm's Problem

- Consider a merged, profit maximizing firm that produces in both markets.
- The merged firm solves

$$\max_{q_c, q_f, \xi} \left\{ p_c q_c + p_f q_f - q_c^2 (1 + (1 - \xi)^2) - q_f^2 (1 + \xi^2) \right\}.$$

12.8.1. The Merger Solution

- The merged firm chooses

$$\begin{aligned} 5 &= 2q_c (5 + (1 - \xi)^2) \\ 2 &= 2q_f (1 + \xi^2) \\ q_c^2 2 (1 - \xi) &= q_f^2 2 \xi \end{aligned}$$

- This gives approximately $\xi = 0.17$, $q_c = 0.44$, and $q_f = 0.97$.

12.8.2. The Merger Solution Welfare

- The externality is internalized.
- The river is not dried out ($\xi \approx 17\%$).
- Cotton production is reduced (**0.44** instead of $\frac{1}{2}$).
- Fish production is increased (**0.97** instead of $\frac{1}{2}$).
- The merged firm's profit is

$$\pi_m \approx 2.07 > \frac{5}{4} + \frac{1}{2} = 1.75$$

- The profit of the merged firm is greater than the sum of the profits of the market with two firms
- This shows that the market solution is not efficient.

12.9. Application: What Happened in the Aral Sea?

- The model shows that internalizing the externality is economically more efficient than the market solution.
- In the case of the Aral Sea, the externality was not internalized, despite that the Soviet Union's economy was centrally planned.
- Why did this happen?
- Did we account for all the costs in the model? Did the Soviet regime do so?

12.10. Other Ways to Obtain Efficient Allocations

- We have seen that internalizing the production externality gives Pareto efficient allocations.
- Any property rights issues are solved when the firms merge.
- Are there other ways we can reach efficient allocations?
- Two alternative ideas proposed in the literature are
 - (Pigouvian) taxation
 - Enforcing rights for the missing market.

12.11. Efficient Production Through Taxation

- Suppose that the government introduces a tax that the cotton producer has to pay for each percentage point of water it draws from the river.
- The modified profit of the cotton producer is

$$\max_{q_c, \xi} \left\{ p_c q_c - q_c^2 (5 + (1 - \xi)^2) - t \xi \right\}$$

- Now, the cotton producer would like to choose q_c and ξ such that

$$\begin{aligned} 5 &= 2q_c (5 + (1 - \xi)^2) \\ t &= q_c^2 2 (1 - \xi) \end{aligned}$$

- How should the government choose t to make production economically efficient?

12.11.1. Choosing the Right Tax

- In principle, the government can calculate the optimal $q_f^* = 0.97$ and $\xi^* = 0.17$ by solving merged firm's problem, and set

$$t = (q_f^*)^2 2 \xi^*.$$

- This enforces

$$q_c^2 2 (1 - \xi) = t = (q_f^*)^2 2 \xi^*,$$

and leads to Pareto efficient allocations.

- In practice, calculating the q_f^* and ξ^* is very difficult.

- It requires information that governments typically do not possess.
- The Pigouvian taxation solution is either impossible or at best impractical.

12.12. Enforcing a Market for the Externality

- Another approach is to enforce property rights for the missing market.
- The government legislates and supervises a market in which the firm(s) receiving the externality can sell the rights to produce the externality to the firm(s) emitting it.
- E.g., in the fishery-cotton example, the government creates a water drain market and gives the right to the fishery to sell a draining percentage to the cotton firm.
- Let r be the price per percentage unit of drained water. The modified profit of the cotton producer is

$$\max_{q_c, \xi} \{ p_c q_c - q_c^2 (5 + (1 - \xi)^2) - r\xi \}$$

- With first order conditions

$$\begin{aligned} 5 &= 2q_c (5 + (1 - \xi)^2) \\ r &= q_c^2 2(1 - \xi) \end{aligned}$$

12.12.1. How Much Water Should the Fishery Sell?

- With the new market, the fishery solves

$$\max_{q_f, \xi} \{ p_f q_f - q_f^2 (1 + \xi^2) + r\xi \}.$$

- The fishery chooses

$$\begin{aligned} 2 &= 2q_f (1 + \xi^2) \\ r &= q_f^2 2\xi \end{aligned}$$

12.12.2. Feasibility of Efficiency

- Combining the fishery and cotton necessary conditions for ξ gives

$$q_c^2 2(1 - \xi) = q_f^2 2\xi.$$

- This is the same condition as in the merged firm's problem.
- Enforcing the missing market is economically efficient.
- However, practically it is not always feasible to regulate the missing market.
- E.g., emitting and receiving firms may operate in different countries.

12.13. Current Field Developments

- Green growth and circular economy are very high on the European policy agenda.
 - In the "Europe 2020" strategy (published in 2010) it was discussed as sustainable, green growth.
 - In 2021, discussions for a new plan termed "A European Green Deal" had started.
- Many economists agree that carbon taxes are the most efficient way to tackle climate change ([Climate Leadership Council 2019](#)).
- There are many difficulties dealing with environmental externalities at an international level.
- The Paris Agreement (2015) is an international treaty on climate change signed by 193 states.

12.14. Concise Summary

- Externalities lead to economically inefficient allocations when property rights cannot be enforced.
- Externalities are very relevant in the analysis of environmental issues.
- If there is a way to restructure production so that the externality is internalized, the market solution can become efficient.
- Essentially internalizing the externality bypasses the problem of non-enforceable property rights.
- The economic literature also proposes other solutions based on
 - taxation (see e.g., carbon tax discussion) and
 - enforcing property rights for the missing market.

12.15. Further Reading

- Varian ([2010, secs. 35.3-7](#))
- CORE Team ([2017, secs. 12.1, 12.4](#))

12.16. Mathematical Details

12.16.1. Production Externalities

Suppose there are two markets with one profit maximizing firm in each of them. The emitter is a firm in the first market producing an externality that negatively affects the production cost of the second market. The receiver is a firm in the second market that cannot control the externality, yet this externality positively affects its production cost.

1. The emitter's problem

The emitter solves

$$\max_{q_e, \xi} \{p_e q_e - c_e(q_e, \xi)\}.$$

The externality, denoted by ξ , is not priced in the market. The emitter controls the level of externality, and its cost function is decreasing in the externality.

2. The receiver's problem

The receiver solves

$$\max_{q_r} \{p_r q_r - c_r(q_r, \xi)\}.$$

The receiver does not control the level of externality, and its cost function is increasing in the externality.

3. The decentralized solution

For interior solutions, the emitter produces at the point that solves

$$\begin{aligned} p_e &= \frac{\partial c_e}{\partial q_e}(q_e, \xi) \\ 0 &= \frac{\partial c_e}{\partial \xi}(q_e, \xi) \end{aligned}$$

The receiver produces at the point that solves

$$p_r = \frac{\partial c_r}{\partial q_r}(q_r, \xi)$$

The emitter ignores the cost that the externality induces to the receiver.

4. The centralized solution

How can the externality be internalized? Consider a merged, profit maximizing firm that produces in both markets. The effect of the production externality of the emitting on the receiving production process is taken into account.

The merged firm solves

$$\max_{q_e, q_r, \xi} \{p_e q_e + p_r q_r - c_e(q_e, \xi) - c_r(q_r, \xi)\}.$$

For interior solutions, the merged firm produces at quantity levels that solve

$$\begin{aligned} p_e &= \frac{\partial c_e}{\partial q_e}(q_e, \xi) \\ p_r &= \frac{\partial c_r}{\partial q_r}(q_r, \xi) \\ \frac{\partial c_e}{\partial \xi}(q_e, \xi) &= -\frac{\partial c_r}{\partial \xi}(q_r, \xi) \end{aligned}$$

The first two conditions are also present in the non merged firms' case. The third condition replaces the second optimization condition of the emitter's problem in the decentralized solution. This condition incentivizes the merged firm to take into account the effects of the externality in the second production process.

5. Non efficiency

The market outcome is not Pareto efficient in the presence of externalities whenever $\frac{\partial c_r}{\partial \xi}(q_r, \xi) \neq 0$ (why?). The emitter tends to produce more than the efficient output at the cost of producing greater externalities. The receiver tends to produce less than the efficient output due to the presence of more than the efficient level of externalities in its production process.

12.17. Exercises

12.17.1. Group A

- Consider two monopolistic markets intermingled by an externality. In the first market, the firm's profit as a function of output is $\pi_1(q_1) = 48q_1 - q_1^2$. The output of the first monopolist affects the profit of the monopolist in the second market as an externality. The second firm's profit as a function output is $\pi_2 = (60 - q_1)q_2 - q_2^2$.

- Suppose that each firm independently maximizes its profit. Calculate the optimal quantity and profit of each market. Moreover, calculate the total profit for both markets.
- Suppose that the firm in market **1** is not allowed to produce any output. Calculate the optimal quantity and profit in market **2**.
- Suppose that firm **1** has to pay a transfer to firm **2**, equal to the damages caused by the production externality. How do the profit functions of the two firms change? Calculate the optimal quantity and profit of each market. Calculate also the total profit in both markets.
- Suppose that the two firms merge. What is the resulting profit function of the merged firm? Calculate the optimal quantity and profit.
- Compare the profits of the above cases. Which case is the most efficient?

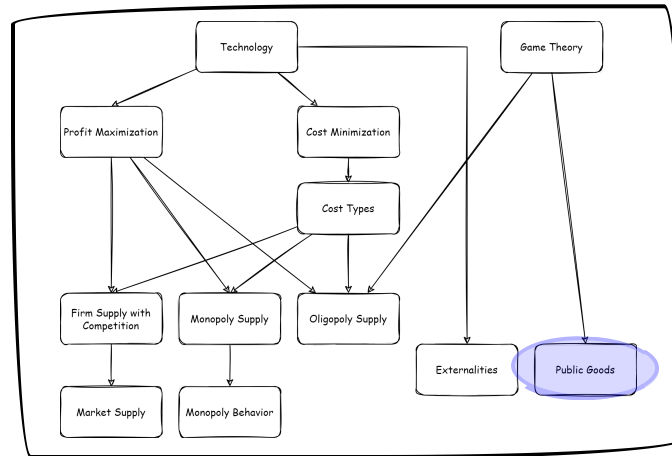
13. Public Goods

13.1. Context

- Free markets of regulation? Externalities can lead to inefficient economic allocations. One remedy in such occasions is for the government to intervene and define a clear structure (new market) in which property rights can be traded.

- Nevertheless, when property rights are non-excludable and should be split among more than one interested party, coordination difficulties become severe and attempts to organize (missing) markets can completely fail.
 - What characteristics lead to market provision failures?
 - What are the welfare implications of such failures?
 - Are there alternative production plans that can be used to produce such products and services?

13.2. Course Structure Overview



13.3. Lecture Structure and Learning Objectives

Structure

- Open Source Software (Case Study)
- Basic Concepts
- A Public Good Game Example
- Welfare Comparisons
- Application: Predictions in the Open Source Software case
- Current Field Developments

Learning Objectives

- Explain the concept of public goods and illustrate why they are relevant.
- Introduce and explain the free-riding problem.
- Explain the difficulties arising in the provision of public goods.
- Illustrate the free-riding problem using a public good game.
- Explain why open-source software is so prevalent today.

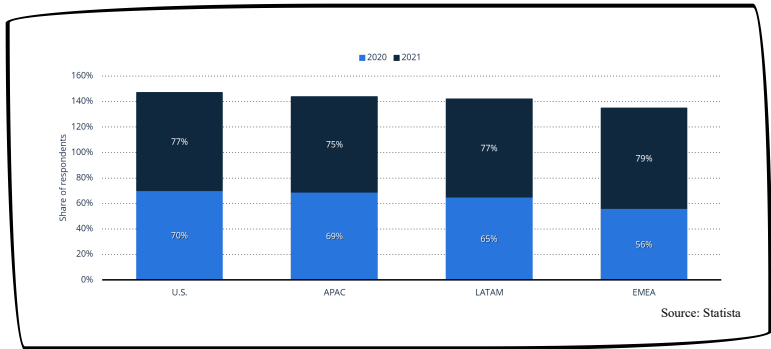
13.4. Open Source Software

- Much of the modern software is free and/or open source.
- Everyone can download and use the software without paying for it.
- In many cases, the source code is also distributed using permissive licenses that
 - allow users to modify the source according to their needs,
 - redistribute (and even sell) the code.
- Usually, there is no direct payment for the developers of the code.



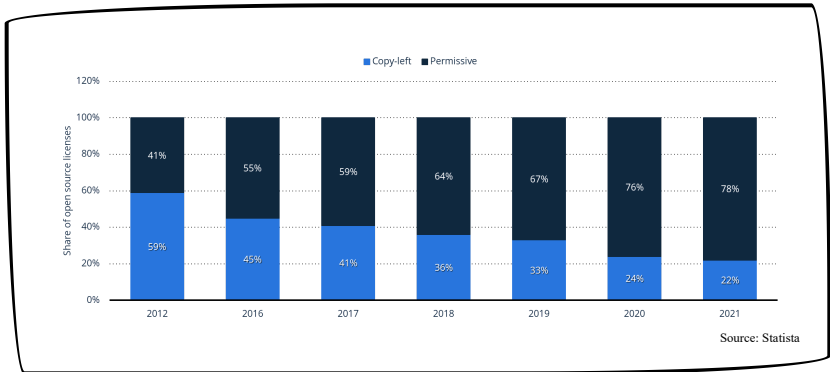
13.4.1. Enterprises Using Open Source for It Infrastructure Modernization Worldwide in 2020 & 2021, by Region

- Most of the digital software infrastructures use at least to some extent open source software.
- The time trend of using open software is positive.



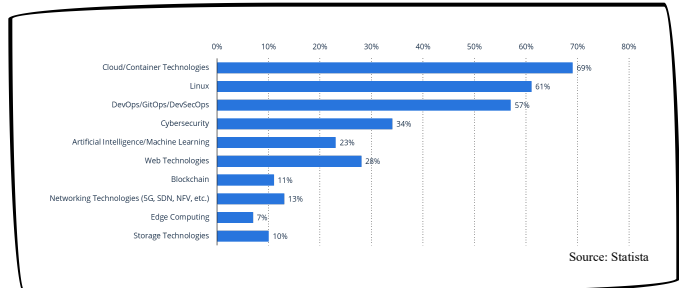
13.4.2. Distribution of Permissive & Copyleft Open Source Licenses Worldwide From 2012 to 2021

- Companies distribute software products with permissive licenses.
- The share of permissive products almost doubled during the last ten years.



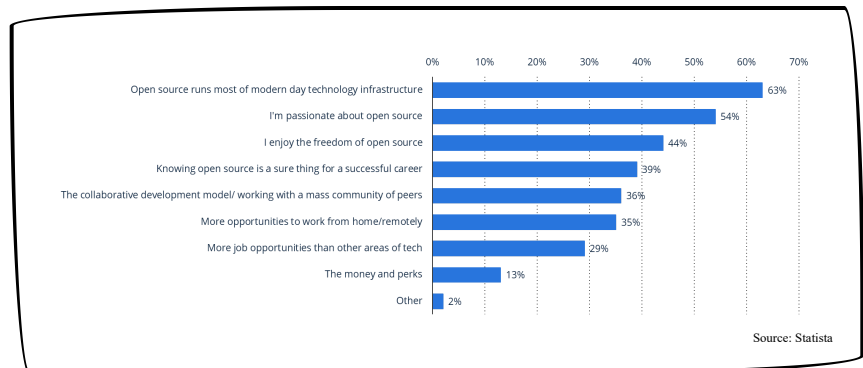
13.4.3. Most Important Open Source Skills for Professionals According to Hiring Managers, as of 2022

- Managers' have endorsed the open-source software model today.
- Although might be permitted by its license, setting a price for open-source cannot exclude non-paying consumers from using it.
- Open-source usage from one company does not reduce the capacity of other companies to use the same software.



13.4.4. Reasons Why Professionals Worldwide Choose Careers in Open Source as of 2022

- Industry culture (status quo) affects developers' decisions.
- Ideals and beliefs play a significant role.
- There is no market system driving development and collaboration.



13.5. Excludability and Rivalry

- A **public good** is a commodity or service that is non-excludable and non-rivalrous.
- A good (or a service) is **non excludable** if the consumption of the good cannot be limited to only paying customers.
- A good (or a service) is **non rivalrous** if the consumption of the good from an agent does not reduce the ability of others to consume it.

13.5.1. Excludability and Rivalry Examples

	Excludable	Non Excludable
Rivalrous	apples, oranges	natural resources
Non Rivalrous	theaters, concerts	open-source software, public television

13.5.2. Public and Other Goods Examples

	Excludable	Non Excludable
Rivalrous	private goods	common goods
Non Rivalrous	club goods	public goods

- A **private good** is a commodity or service that is excludable and rivalrous.
- A **common good** is a commodity or service that is non-excludable and rivalrous.
- A **club good** is a commodity or service that is excludable and non-rivalrous.
- Bonus: What is the difference between public goods and externalities?

13.6. Market Collapse

- The combination of non-excludability and non-rivalry can make the market-based provision of public goods impossible.
- Non altruistic agents avoid contributing to the public good production and attempt to free-ride.
- This is typically Pareto inefficient.

13.7. Free Riding

- The **free riding** problem occurs whenever there are economic agents who benefit from using resources either without paying at all or by underpaying for acquiring them.
- Examples:
 - All citizens consume the same level of national defense, irrespective of their individual contributions to the national budget.
 - All ships benefit from a lighthouse, irrespective of paying for its construction.

13.7.1. Free Riding Game

- Two roommates consider subscribing to Netflix (say **15** Euros per subscription).
- Each roommate values the service **10** Euros.

		Roommate B	
		Buy	Not Buy
Roommate A	Buy	-5, -5	-5, 10
	Not Buy	10, -5	0, 0

- No one ends up buying the subscription.
- Bonus: What is the difference with prisoners' dilemma?

13.8. The Public Good Game

- Suppose that there are **2** firms in a market.
- Each firm has a budget equal to **100** Euros.
- They decide how much to contribute to the production of a network infrastructure (say x_1 and x_2).
- The sum of the contributions is used in the production of the infrastructure.

$$q = f(x_1, x_2) = 3 \frac{x_1 + x_2}{2}.$$

- The produced output (infrastructure capacity) is equally split.

13.8.1. The Firm's Problem

- The firms maximize their profits

$$\pi_i(x_1, x_2) = 3 \frac{x_1 + x_2}{4} + 100 - x_i.$$

13.8.2. The Market Solution

- The profit of firm **1** is strictly decreasing in its contribution.

$$\frac{\partial \pi_1}{\partial x_1}(x_1, x_2) = \frac{3}{4} - 1 < 0.$$

- Therefore, $x_1 = 0$ and

$$\pi_1(0, x_2) = 3\frac{x_2}{4} + 100.$$

- Since the game is symmetric, the same argument is valid for agent 2.
- Therefore the Nash equilibrium of the game is $x_i = 0$, with payoffs $\pi_i = 100$ for every agent i .
- The total welfare, in this case, is equal to $\pi_1 + \pi_2 = 200$.
- Both firms try to free-ride, and no production takes place.

13.8.3. Efficient Provision of the Public Good

- The total welfare in the market is determined by (why?)

$$W(x) = 3\frac{x}{2} + 200 - x.$$

- The total welfare is increasing in x . Therefore, it is maximized for $x = 200$.
- The maximized total welfare is $W(200) = 300$.
- This solution is Pareto dominates the market solution $W(200) > \pi_1 + \pi_2$.

13.9. Discussion: Free and Open Source Software

- Is open-source software a public good?
- Can open-source software be priced?
- Why do people contribute and use open-source software?

13.10. Current Field Developments

- Public goods games are used by economists to study cooperation and coordination ([Takeuchi and Seki 2023](#))
- Digital public goods have a central role in the discussion of achieving a sustainable economy.
- At the same time, they create a series of new challenges:
 - Should the AI models be democratized?
 - What are the security implications of using Open Source Software in the governments' infrastructures?

13.11. Concise Summary

- Free riding can lead to economically inefficient allocations.
- A common reason that free riding arises is the presence of public goods.
- Public goods can lead to a complete production shutdown.
- State provision is the most common way of supplying public goods.

13.12. Further Reading

- Varian ([2010, secs. 37.1-4](#))
- CORE Team ([2017, sec. 12.5](#))

13.13. Mathematical Details

13.13.1. Public Goods

A public good is a good that is non-excludable and non-rivalrous. The consumption of a commodity is non-rivalrous when its consumption from an agent does not affect the availability of the commodity for other agents. The consumption of a commodity is non-excludable when it is impossible to exclude agents from its consumption.

13.13.2. The Public Good Game

Suppose that there are n agents in a market. Each agent has a budget equal to B . She decides how much she contributes to the production of a public good. The sum of the contributions of all players is used as input in a linear production technology that scales them by $1 < A < n$. The production output is split equally among all agents. The agent payoff is determined by

$$\pi_i(x_1, \dots, x_n) = \frac{A \sum_{j=1}^n x_j}{n} + B - x_i.$$

13.13.3. The Market Solution

The payoff of each agent is strictly decreasing in her own contribution, i.e.,

$$\frac{\partial \pi_i}{\partial x_i}(x_1, \dots, x_n) = \frac{A}{n} - 1 < 0.$$

Therefore, the payoff is maximized for $x_i = 0$, resulting in

$$\pi_i(0, \dots, x_n) = \frac{A \sum_{j \neq i}^n x_j}{n} + B.$$

Since the game is symmetric, the same argument is valid for all agents. Therefore, the Nash equilibrium of the game is $x_i = 0$, with payoffs $\pi_i = B$ for every agent i . The total welfare, in this case, is equal to nB .

13.13.4. The Efficient Provision of the Public Good

The total welfare in the market is determined by

$$W(\mathbf{x}) = A\mathbf{x} + nB - \mathbf{x}$$

The total welfare is increasing in $\mathbf{x}$. Therefore, it is maximized for $\mathbf{x} = nB$. Thus, the total welfare is $\pi = AnB$ at the optimum. Since $A > 1$, this solution Pareto dominates the market solution.

13.14. Exercises

13.14.1. Group A

- Suppose there are two individuals, **1** and **2**, each consuming one private good in the amounts $\mathbf{x}_1$ and $\mathbf{x}_2$, respectively. The price of this private good is $\mathbf{p}$. In addition, they consume a public good $\mathbf{G}$. The marginal cost of production of $\mathbf{G}$ is equal to one. The amount of the public good is determined by $\mathbf{G} = \mathbf{g}_1 + \mathbf{g}_2$, where $\mathbf{g}_1$ and $\mathbf{g}_2$ are the individual contributions of the players. Individual $\mathbf{i}$'s preferences are represented by a Cobb-Douglas utility function $u(\mathbf{x}_i, \mathbf{G}) = \mathbf{x}_i^\alpha \mathbf{G}^\beta$. Both players have a budget of $\mathbf{B}$.
 - Explain why in this case, $\mathbf{G}$ can be considered a public good.
 - Set up individual $\mathbf{i}$'s budget constraint.
 - Find individual $\mathbf{i}$'s best response contribution $\mathbf{g}_i$.
 - Compute the Nash equilibrium for the contributions $\mathbf{g}_1$ and $\mathbf{g}_2$.
 - Does the Nash equilibrium allocation entail more or fewer resources directed to the public good than the Pareto efficient allocation?
 - Argue that the Nash equilibrium is not Pareto-efficient.

13.14.2. Group B

- Consider a public good game with two players whose payoffs are determined by $\pi_1(\mathbf{q}) = (1 + \mathbf{q})(\mathbf{B}_1 - \mathbf{q}\mathbf{x}_1)$ and $\pi_2(\mathbf{q}) = (2 + \mathbf{q})(\mathbf{B}_2 - \mathbf{q}\mathbf{x}_2)$. In these expressions, $\mathbf{x}_1$ and $\mathbf{x}_2$ represent the player specific, fixed contributions to producing the public good, and $\mathbf{q}$ the public good output. Players have a binary choice concerning the public good; they can commonly choose $\mathbf{q} = 1$ and purchase the public good, or $\mathbf{q} = 0$ and abstain from it. Player 1 has budget $\mathbf{B}_1$, while player 2 has budget $\mathbf{B}_2$.
 - What is the maximum amount that player 1 is willing to contribute to the public good?
 - What is the maximum amount that player 2 is willing to contribute to the public good?
 - Suppose that $\mathbf{B}_1 = 100$ and $\mathbf{B}_2 = 75$. By producing the public good, the players have a Pareto improvement over not producing it, as long as the production cost of the public good is no greater than $\hat{\mathbf{x}}$. Find $\hat{\mathbf{x}}$.
 - Find a combination of specific contributions $\mathbf{x}_1$ and $\mathbf{x}_2$ whose sum is less than $\hat{\mathbf{x}}$, but for which letting the players individually maximize their payoffs does not result in a Pareto efficient allocation.

14. Mathematical Appendix

The mathematical appendix contains some remainders of useful mathematical results of general interest, remainders of frequently used calculus rules, and frequently used symbols of the material.

14.1. Used Limit Results

- Let $\mathbf{f}$, $\mathbf{g}$, $\mathbf{h}$ be functions.
- Let $\mathbf{c}$, $\mathbf{L}$ be real numbers, and $\alpha > 0$.

14.1.1. Continuous Mapping Theorem

- If $\mathbf{f}$ is continuous at $\mathbf{x}_0$,
- then $\lim_{\mathbf{x} \rightarrow \mathbf{x}_0} \mathbf{f}(\mathbf{x}) = \mathbf{f}(\mathbf{x}_0)$

14.1.2. Squeeze Theorem

- If $\lim_{\mathbf{x} \rightarrow \mathbf{c}} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{x} \rightarrow \mathbf{c}} \mathbf{h}(\mathbf{x}) = \mathbf{L}$ and $\mathbf{f}(\mathbf{x}) \leq \mathbf{g}(\mathbf{x}) \leq \mathbf{h}(\mathbf{x})$,
- then $\lim_{\mathbf{x} \rightarrow \mathbf{c}} \mathbf{g}(\mathbf{x}) = \mathbf{L}$.

14.1.3. L'Hôpital's Rule

- If $\lim_{\mathbf{x} \rightarrow \mathbf{c}} \mathbf{f}(\mathbf{x}) = \lim_{\mathbf{x} \rightarrow \mathbf{c}} \mathbf{g}(\mathbf{x}) = 0$ or $\pm \infty$, and $\mathbf{g}'(\mathbf{x}) \neq 0$, and $\lim_{\mathbf{x} \rightarrow \mathbf{c}} \frac{\mathbf{f}'(\mathbf{x})}{\mathbf{g}'(\mathbf{x})}$ exists,
- then $\lim_{\mathbf{x} \rightarrow \mathbf{c}} \frac{\mathbf{f}(\mathbf{x})}{\mathbf{g}(\mathbf{x})} = \lim_{\mathbf{x} \rightarrow \mathbf{c}} \frac{\mathbf{f}'(\mathbf{x})}{\mathbf{g}'(\mathbf{x})}$.

14.1.4. Limits of Powers

$$\begin{aligned} \lim_{\mathbf{x} \rightarrow \mathbf{x}_0} \mathbf{c} &= \mathbf{c} \\ \lim_{\mathbf{x} \rightarrow \infty} \mathbf{x}^\alpha &= \infty \\ \lim_{\mathbf{x} \rightarrow \infty} \mathbf{x}^{-\alpha} &= \lim_{\mathbf{x} \rightarrow \infty} \frac{1}{\mathbf{x}^\alpha} = 0 \\ \lim_{\mathbf{x} \rightarrow \mathbf{c}} \mathbf{x}^\alpha &= \mathbf{c}^\alpha \\ \lim_{\mathbf{x} \rightarrow \mathbf{c}} \mathbf{x}^{-\alpha} &= \mathbf{c}^{-\alpha} \end{aligned}$$

14.1.5. Limits of Exponentials

$$\lim_{x \rightarrow \infty} \alpha^x = \begin{cases} \infty & \alpha > 1 \\ 1 & \alpha = 1 \\ 0 & \alpha < 1 \end{cases}$$

$$\lim_{x \rightarrow \infty} \alpha^{-x} = \lim_{x \rightarrow \infty} \frac{1}{\alpha^x} = \begin{cases} 0 & \alpha > 1 \\ 1 & \alpha = 1 \\ \infty & \alpha < 1 \end{cases}$$

14.2. Used Derivative Results

- Let f, g be functions.
- Let c be any real number, and $\alpha > 0$.

14.2.1. Definition of the Derivative

$$\bullet \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = f'(x)$$

14.2.2. Inverse Function Theorem

- If f is a continuously differentiable function with $f'(x_0)$ at the point x_0 , then
 - f is invertible in a neighborhood of x_0 ,
 - the inverse is continuously differentiable, and
 - the derivative of the inverse function at $y_0 = f(x_0)$ is the reciprocal of the derivative of f at x_0 , i.e.

$$(f^{-1})'(y_0) = \frac{1}{f'(x_0)} = \frac{1}{f'(f^{-1}(y_0))}.$$

14.2.3. Implicit Function Theorem

- Let f be a continuously differentiable function of two variables and a $(\hat{x}_1, \hat{x}_2)$ be a point so that $f(\hat{x}_1, \hat{x}_2) = \bar{f}$. If $f_{x_2}(\hat{x}_1, \hat{x}_2) \neq 0$, then
 - there is a neighborhood of $(\hat{x}_1, \hat{x}_2)$ and an implicit function g such that

$$f(x_1, g(x_1)) = c$$

for all x_1 in the neighborhood, and

- the derivative of the implicit function is given by

$$g'(x_1) = \frac{dx_2}{dx_1} = -\frac{f_{x_1}(x_1, g(x_1))}{f_{x_2}(x_1, g(x_1))}$$

for all x_1 in the neighborhood.

14.2.4. Basic Differentiation Rules

$$\begin{aligned} \frac{d}{dx}(f+g) &= \frac{df}{dx} + \frac{dg}{dx} \\ \frac{d}{dx}(c \cdot f) &= c \cdot \frac{df}{dx} \\ \frac{d}{dx}(f \cdot g) &= f \cdot \frac{dg}{dx} + g \cdot \frac{df}{dx} \\ \frac{d}{dx}\left(\frac{f}{g}\right) &= \frac{-f \cdot \frac{dg}{dx} + g \cdot \frac{df}{dx}}{g^2} \\ \frac{d}{dx}[f(g(x))] &= \frac{df}{dg} \cdot \frac{dg}{dx} = f'(g(x)) \cdot g'(x) \\ \frac{d}{dx}\left(\frac{1}{f}\right) &= -\frac{f'}{f^2} \end{aligned}$$

14.2.5. Differentiation of Powers

$$\begin{aligned} \frac{d}{dx}(c) &= 0 \\ \frac{d}{dx}\sqrt{x} &= \frac{1}{2\sqrt{x}} \\ \frac{d}{dx}x &= 1 \\ \frac{d}{dx}\frac{1}{x} &= -\frac{1}{x^2} \end{aligned}$$

And more generally

$$\frac{d}{dx} x^c = cx^{c-1}$$

14.2.6. Differentiation of Exponentials and Logarithms

$$\begin{aligned}\frac{d}{dx} e^x &= e^x \\ \frac{d}{dx} a^x &= a^x \ln(a) \\ \frac{d}{dx} \ln(x) &= \frac{1}{x}\end{aligned}$$

14.3. Elasticity

- For a differentiable function $f \neq 0$, the elasticity is defined by

$$e^f(x) = f_x(x) \frac{x}{f(x)} = \frac{df(x)}{dx} \frac{x}{f(x)}.$$

- For appropriate functions $y = f(x)$, by the inverse function theorem, we also have

$$e^f(x) = \frac{1}{f_y^{-1}(y)} \frac{x}{f(x)} = \frac{1}{\frac{dx}{dy}} \frac{x}{f(x)}.$$

14.4. Table of Integrals

- Let f, g be differentiable functions and f', g' be their derivatives.
- Let c be any real number, and $\alpha > 0$.

14.4.1. Definite and Indefinite Integration

- Indefinite integral (is a function)

$$f(x) = \int_0^x f'(z) dz,$$

- also usually denoted as

$$\int f'(x) dx + K,$$

where K is a constant.

- Definite integral (is a number)

$$\int_{x_0}^{x_1} f'(x) dx = f(x_1) - f(x_0).$$

14.4.2. Basic Integration Rules

$$\begin{aligned}\int [f(x) + g(x)] dx &= \int f(x) dx + \int g(x) dx \\ \int cf(x) dx &= c \int f(x) dx \\ \int f(x)g'(x) dx &= f(x)g(x) - \int f'(x)g(x) dx\end{aligned}$$

14.4.3. Integration of Powers

$$\begin{aligned}\int c dx &= cx + K \\ \int x^c dx &= \frac{x^{c+1}}{c+1} + K\end{aligned}$$

14.4.4. Integration of Exponentials

$$\int e^x dx = e^x + K$$

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